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Figure S1: Energy spectrum of antiferromagnetic tetragonal CuMnAs on the conventional k-paths. a with SOC turned off, and **b** with SOC turned on.

Figure S2 iIlÜÚÛÙaÛeÚÛhe?hiddeÕÚ×iÕ×ÖlaÙiáaÛiÖÕʔeffecÛiÕheßagÖÕalCÖBÙ2 [3] being bulk SST-2 class (MSG C_c2/c) but made of SST-5 class FM $CoBr_2$ layers (-sector and -sector in Fig. S2a). The crystal is antiferromagnetically ordered with its magnetic moments collinearly aligned in (001) direction. The two $FeBr_2$ layers are connected by both the and the symmetry which restores the spin degeneracy of the bulk and results in a compensated net spin polarization (Fig. S2b). However, the corresponding spin polarization for the bottom two conduction bands (C1 and C2) projected onto -sector and -sector (hidden spin polarization), shown in Fig. S2c, are non-zero and compensate each other.

Figure S2: Hidden spin polarization from individual ferromagnetic sectors in bulk hexagonal $CoBr_2$ (bulk belonging to SST-2 class with sector belonging to SST-5 class). a crystal structure of antiferromagnetic $CoBr_2$ composed of two ferromagnetic layers with opposite magnetization (indicated by red and blue polyhedra) in the unit cel lyhedra to red. The crystal and magnetic structure for hexagonal $CoBr_2$ used in our DFT calculations are taken from Ref. [3]. The electronic structure and hidden spin polarization are calculated using the PBE+U method with U=3.32 eV, J=0 eV on Co-3d orbits.

Figure S3 illÜÚÛÙaÛeÚÛhe?hiddeÕÚ×iÕ×ÖlaÙiáaÛiÖÕ?effecÛiÕÛeÛÙagÖÕal $Ca_3Ru_2O_7$ [4] being bulk SST-3 class (MSG P_Cna2_1) but made of SST-5 class FM RuO_7 sectors (-sector and -sector in Fig. S3a). The crystal is antiferromagnetically ordered with its magnetic moments collinearly aligned in (010) direction. The two ferromagnetically ordered Ru_2O_7

Figure S5: Hidden spin polarization from individual antiferromagnetic sectors in bulk MnS_2 (bulk belonging to SST-3 class with sector belonging to SST-4 class). a crystal structure of antiferromagnetic MnS_2 composed of two antiferromagnetic sectors with opposite magnetic ordering (indicated by red and blue polyhedra) in the unit cell. The two layers are referred as sector- and sector- , respectively; b spin degenerate bands of MnS_2. c Hidden spin polarization from each individual sector of the highest two valence bands (V1 and V2) on k-plane. The up and down spins are mapped to the color from blue to red. The crystal and magnetic structure for tetragonal MnS2 used in our DFT calculations are taken from Ref. [6]. The electronic structure and hidden spin polarization are calculated using the PBE+U method with U=5.0 eV, J=0 eV on Mn-3d orbits.

Figure S6 illustrates how SST-4 class monolayers (wTm SS) reE}>TJETQ0.000009 2 0 612 792 reW*nBT/F8 12 polarization in a bilayer with -asymmetric subsets that compensate each other. The SS magnitude depends on the intensity of the external electric field. Different from the relativistic Rashba and Dresselhaus SS that require the SOC, the electric field induces in the SST-1 FeSe bilayer a non-zero SS even in the absence of SOC (Fig. S6c).

Figure S6: Geometry an