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Search, Heterogeneity, and Optimal Income Taxation

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Working Paper

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Abstract

D

. N fi,

M (1982), H (1990) A (1999)

). H ,

(2001) ,

fi ff

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ff

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2 Model

2.1 The matching technology

, 1 – ff

. A M P (1999) ,

. H , P (1996) Bl D -

2.2 Output sharing

[illegible]

2.4 Private expected utility functions

fi , , $v_H q_H =$ $_m q_m$.

k

$$U_k = -c_w(\ _k) + \ _k \ M(\ ) \ \frac{v_H q_H}{_m v_m q_m} \ _{kH} y_{kH} + \frac{v_L q_L}{_m v_m q_m} \ _{kL} y_{kL} \ + (1 - M(\ ))0 \ + (1 - \ _k)0$$

$$U_k = -c_w(\ _k) + \ _k M(\ ) E_{(m)} \ _{km} y_{km}; \quad (5)$$

$E_{(m)}$ ll

. A , , $c(\ )$, . O , ,

$M(\ )$,

. I ,

$1 - M(\ )$, , ,

. I ,

(

3 Optimal search intensity and market ine ciencies

3.1 Social Optimum

A [illegible] :

$$W = {}_{\delta,v} \quad {}_{k} l_k U^k + {}_{m} q_m V^m$$

$$. \ . \quad {}_k \geq 0; \quad v_m \geq 0.$$

(1), (2), (5), (6), -91.21

$$E_{(m)}y_{km} - (1 - \quad)E_{(k)}E_{(m)}y_{km} = \quad E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}:$$

A ... (1 ...) ... ($E_{(k)}E_{(m)}y_{km}$), ... ff ... , $E_{(m)}y_{km}$, ... , $E_{(k)}E_{(m)}y_{km}$. ...

... :

... 1 . *I* ...

$$\begin{aligned} c'_{\pi}(\bar{v}_H) &= \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km} \\ c'_{\pi}(\bar{v}_L) &= \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} \end{aligned} \quad \left| \begin{array}{l} \bar{\ } \lesseqgtr 1; \\ \bar{v}_H > 0;\ \bar{v}_L > 0 \end{array} \right. ; \tag{13}$$

$$\begin{aligned} c'_{\pi}(\bar{v}_H) &= \frac{M(\ )}{} E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km} \\ c'_{\pi}(0) &\geq \frac{M(\ )}{} E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} \end{aligned} \quad \left| \begin{array}{l} \bar{\ } \lesseqgtr 1; \\ \bar{v}_H > 0;\ \bar{v}_L = 0 \end{array} \right. : \tag{14}$$

3.2 Decentralized equilibrium

A ... k ...

$$\max_{\delta_k} \quad U_k = -c_w(\ _k) + \ _kM(\ )E_{(m)} \ _{km}y_{km} \quad \text{s.t.} \ _k \geq 0; \tag{15}$$

... $C_k = c_w(\ _k)$ ... , ... $B_k = \ _kM(\ )E_{(m)} \ _{km}y_{km}$ ... fi ... .

K $\ _k$ ^{2}

$$-c_w'(\ \ _k) + M(\ \ )E_{(m)}\ \ _{km}y_{km} \le 0$$

$$_k \ge 0 \qquad (16)$$

$$(-c_w'(\ \ _k) + M(\ \ )E_{(m)}\ \ _{km}y_{km})\ \ _k = 0;$$

$_k$, . $M(\ )$, . F ,

ff , ff .

I -

. $_{LL} \neq 0$ ($_{LH} = 0$), $y_{km} > 0$,

fi (06)2 158.730 ( ) -395. 4.9812163(0),)-373( )-333(( )-350(5. 4

k (m)

$_H M(\ )E_{(m)} w_{Hm}$.

$\left(_L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}} \right. M(\ )E_{(m)} w_{Hm} = {}_L M(\ )E_{(m)} w_{Lm}$, $\left(_L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}} \right.$

$_L M(\ )E_{(m)} w_{Lm}$ [3].

$_H M(\ )E_{(m)} w_{Hm}$ $_L M(\ )E_{(m)} w_{Lm}$,

^w_H, $_H M(\ )E_{(m)} w_{Hm}$, ^w_L, $_L M(\ )E_{(m)} w_{Lm}$.

[13]

. O fi

.

4. *I*

$$
\left.\begin{aligned}
c'_w(\ \ _k) &= M(\ \ )(1 - \ \ ^w_k)w_k \\
c'_\pi(v_m) &= \frac{M(\ \ )}{\ }(1 - \ \ ^\pi_m)\ \ _m
\end{aligned}\right| \begin{aligned} &\lesseqgtr 1; \\ &_k > 0;\ v_m > 0 \end{aligned} \quad ; \tag{22}
$$

$$
\left.\begin{aligned}
c'_w(0) &\geq M(\ \ )(1 - \ \ ^w_L)w_L \\
c'_\pi(0) &\geq \frac{M(\ \ )}{\ }(1 - \ \ ^\pi_L)\ \ _L
\end{aligned}\right| \begin{aligned} &\lesseqgtr 1; \\ &_L = 0;\ v_L = 0 \end{aligned} \quad : \tag{23}
$$

4.1 Characterizing externalities through Pigou taxes

, P $(\check{\ }^w_H; \check{\ }^w_L)$, P $(\check{\ }^\pi_H; \check{\ }^\pi_L)$. L ( ) $\check{R}$, P , , LS:

$$
\check{R} = (\ \ _k\ \ l)M(\ \ ) \ \ \frac{_H l_H}{_k\ \ l}\check{\ }^w_H w_H + \frac{_L l_L}{_k\ \ l}\check{\ }^w_L w_L + \frac{v_H q_H}{_m\, vq}\check{\ }^\pi_H\ \ _H + \frac{v_L q_L}{_m\, vq}\check{\ }^\pi_L\ \ _L
$$

$$
0 = \check{R} - \sum_k l_k + \sum_m l_m\ \ LS;
$$

$(\ \ _k\ \ l)M(\ \ ) = N$.

k

$$
U_k = -c_w\left(\frac{z^w_k}{M(\ \ )\,w_k}\right) + LS + (1 - \check{\ }^w_k)z^w_k \tag{24}
$$

, LS, ff

. fi ,

c'_w

(P 5)

. CR ,

;

,

. .

- , P

,

.

- , ,

ff . I

fi , fi , L ,

.

N , ,

ff $(g = 0)$.

,

. I

$y_{LL} > 0$,

4.2 Optimal income taxes with positive government revenue

$$W = \sum_k l_k U^k + \sum_m q_m V^m ;$$

(1), (2), (5), (6), $w_k = E_{(m)} \; _{km} y_{km}$, $z^w_k = \; _k M(\;) w_k$, $_m = E_{(k)}(1 - \; _{km}) y_{km}$, $z^\pi_m = v_m \frac{M\,\theta}{\theta} \; _m$ 4,

$$W = \sum_k l_k \; -c_w \; \frac{z^w_k}{M(\;)\,w_k} \; + \sum_m q_m \; -c_\pi \; \frac{z^\pi_m}{\frac{M\,\theta}{\theta} \; _m} \; + (\sum_k l) M(\;) \, E_{(k)} E_{(m)} y_{km};$$

$(\sum_k l) M(\;) = N$

R,

$$R \le (\sum_k l) M(\;) \; \frac{_H l_H}{\sum_k l} \; ^w_H w_H + \frac{_L l_L}{\sum_k l} \; ^w_L w_L + \frac{v_H q_H}{\sum_m vq} \; ^\pi_H \; _H + \frac{v_L q_L}{\sum_m vq} \; ^\pi_L \; _L \; ; \qquad (30)$$

$(\sum_k l) M(\;) = M$ (30),

[17]

$$W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ ) w_k}\right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{\frac{M\ \theta}{\theta}\ _m}\right)$$

$$+(\ \ _k\ l)M(\ )\ \frac{_H l_H}{_k\ l}(1 - \ _H^w)w_H + \frac{_L l_L}{_k\ l}(1 - \ _L^w)w_L$$

$$+\frac{v_H q_H}{_m\ vq}(1 - \ _H^\pi)\ \ _H + \frac{v_L q_L}{_m\ vq}(1 - \ _L^\pi)\ \ _L + R:$$

[illegible] (22) [illegible] C [illegible] (1978), [illegible] D [illegible] (1998) [illegible]

[illegible] H [illegible]

[illegible] L [illegible] 7 [illegible] A [illegible] . E [illegible] (22) [illegible]

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ ) w_k}\right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{M\ \theta}\right.$$

[illegible] P [illegible] 8 [illegible] A [illegible]. [illegible]

[illegible] $''^w_k = 0$ [illegible] $''^\pi_m = 0$. [illegible] (33). [illegible] H [illegible] P [illegible] 5 [illegible] A [illegible] A). H [illegible].

[illegible] $''^\pi_m = \infty$, [illegible] $''^\pi_m = \infty$, [illegible].

4.2.2 [illegible]

[illegible] 1 [illegible]

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( ),
(D (1998)). fi
’ (P 11),
, .
P 11 ( )

. A

P 5,

12.

$$i)\ @\ \frac{w_H}{w_L} = @\ \frac{E_{(m)}\ Hm}{E_{(m)}\ Lm} < 0 \quad (36)$$

$$ii)\ @\ \frac{w}{\pi} = @(\ ) < 0 \quad (37)$$

P 11 A . B

P 5).

P 12

B B (2002)

5 Conclusion

(1) "H Effi F ," *I E* , 40, 827-849.

(2002) "O L ," *J E* , 85, 53-97.

(1 4) "Eff B fi F P .

(1) "M Cl ," *J E* , 112, 141-168.

(1) "L - P F : M E ," *E J* , 109, 307-334.

(1) "O R : M R ," *A E* , 68, 756-768.

(1 1) "M C , F Effi ," *J E* , .89, 798-812.

(1 2) "A D M E ," *J E* , .90, 881-894.

(1 2) " D Effi E ," *E* , .49, 217-227.

(1) "O I : A E P O M R ," *A E* , .88, 83-95.

(1) " P ," *E J* , 7, 46-70.

(1 3) "O O P I ," *J E* , 2, 357-376.

(1) "H D P Aff E ? C O E ," M P .

, . (1 0) "Effi I ," J E , 51, 8999.

., . (1 5) "P , , P , D E , .

, . . (1 0) "O Effi M R M ," E , 57, 279-298.

, . (1) " , F , ," E , 32, D E , .

, ., . (1) " P G E P M B ," L E , 3, 6580.

., . (1) "L R , G J B J ," E , C , P .

, ., . (1 3) "

., (1) "L M D Eff ," J E , 33, 457495.

, ., (2000) "A M ," E , 68, 343-369.

, ., (2001) "M , , H " A , 1(2001), I . 1, A . 5.

. (1) "P F E P ?," E , 25, 223264.

. (1) "O P F L M ," L E , 6, 435452.

, . (1) "P Effi O N N E ," I : H E , .2.

, . (1) " - P Effi ," J E , 6, 239262.

, . (1) " , P P D .," I F , 6, 239262.

, . (1) " P F E ," I : , .B. (E .), F C ., M M , L .

Appendices:

A Proofs of the main results

Proof of Corollary 3.

F , . -

, $_1 \neq$. I $_1 >$,

$_H(_1) > _H()$, $_L(_1) > _L()$, $v_H(_1) < v_H()$, $v_L(_1) < v_L()$. H

)$_i$)

CR

. F $1 - \quad =$

$$\check{R} = N \left[\begin{array}{l} \frac{\delta_H l_H}{k\, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} (1 - (1 - \quad)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} (1 - (1 - \quad)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kL} y_{kL} \end{array} \right]$$

$$= N \left[\begin{array}{l} \frac{\delta_H l_H}{k\, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kL} y_{kL} \end{array} \right]$$

$$\check{R} = N \quad 1 - (\quad + \quad)$$

; - , . I (L).□

Proof of Lemma 7.

I , k

$$U_k = \; - c_w (\; {}_k) + \; {}_k M(\;)(1 - \; {}^{w}_{k}) w_k$$
$$= \; - c_w \quad \frac{z^w_k}{M(\;) w_k} \quad + (1 - \; {}^{w}_{k}) z^w_k$$

w_k

$$\frac{@U_k}{@w_k} = -c'_w \frac{1}{M(\;) w_k} \frac{@z^w_k}{@w_k} - c'_w \frac{z^w_k}{M(\;)} \quad - \frac{1}{w^2_k} \quad + \frac{@z^w_k}{@w_k} (1 - \; {}^{w}_{k}) = c'_w \frac{z^w_k}{M(\;) w_k} > 0;$$

E . □

Proof of Proposition 8.

I: $w_k = E_{(m)} w_{km} = E_{(m)} \; {}_{km} y_{km}$

$k = H; L;$ $z_k^w = \; {}_k M(\;) w_k$

$k = H; L;$ ${}_m = E_{(k)} \; {}_{km} = E_{(k)} (1 - \; {}_{km}) y_{km}$ fi

e $m = H; L;$ $z_m^\pi = v_m$ M

Z π π

k

$$\frac{dz_H^w}{z_H^w}\left(\frac{1}{''^w_H} + (1- \quad)\frac{{}_H l_H}{k \quad l}\right) = (1- \quad) \quad E_{(m)}\left(\frac{dz_m^\pi}{z_m^\pi} - \frac{{}_L l_L}{k \quad l}\frac{dz_L^w}{z_L^w}\right) - \frac{d \quad {}^w_H}{1- \quad {}^\pi} \qquad ($$

(45) (46)

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^\pi_H} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \; + \frac{d\tau_L^\pi}{-\tau_L^\pi} \; \frac{v_L q_L}{_m\, vq}''^\pi_L - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1 + \; \frac{v_L q_L}{_m\, vq}''^\pi_L}{\Delta_2} \tag{45}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^\pi_L} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \; - \frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1 + \; \frac{v_H q_H}{_m\, vq}''^\pi_H \; + \frac{d\tau_H^\pi}{-\tau_H^\pi} \; \frac{v_H q_H}{_m\, vq}''^\pi_H}{\Delta_2}; \tag{46}$$

$\Delta_2 = 1 + \; E_{(m)}''^\pi_m$. A (45) (46)

$$E_{(m)} \; \frac{dz_m^\pi}{z_m^\pi} \; = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \; E_{(m)}''^\pi_m - E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2}; \tag{47}$$

(43) (44)

$$E_{(k)} \; \frac{dz_k^w}{z_k^w} \; = \frac{(1- \;)E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi} \; E_{(k)}''^w_k - E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w}}{\Delta_1}: \tag{48}$$

F (47) (48) $E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}$ $E_{(k)}\Big(\frac{dz_k^w}{z_k^w}$

, ,

$$E_{(m)} \; \frac{dz_m^\pi}{z_m^\pi} \; = - \frac{(\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \; + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{49}$$

$$E_{(k)} \; \frac{dz_k^w}{z_k^w} \; = - \frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \; + (\Delta_1 - 1)E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{50}$$

(49) (43) (44), (50) (45) (46) fi /

:

$$\frac{dz_H^w}{z_H^w}\frac{1}{''^w_H} = - \frac{(1- \;) \; (\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \; + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \; \frac{d\tau_L^w}{-\tau_L^w}(1- \;)\frac{\delta_L l_L}{_k\,\delta l}''^w_L - \frac{d\tau_H^w}{-\tau_H^w}\Big(1 + (1- \;)\frac{\delta_L l_L}{_k\,\delta l}''^w_L}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{51}$$

$$\frac{dz_L^w}{z_L^w}\frac{1}{''^w_L} = - \frac{(1- \;) \; (\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \; + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \; - \frac{d\tau_L^w}{-\tau_L^w}\Big(1 + (1- \;)\frac{\delta_H l_H}{_k\,\delta l}''^w_H \; + \frac{d\tau_H^w}{-\tau_H^w}(1- \;)\frac{\delta_H l_H}{_k\,\delta l}''^w_H}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{52}$$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^\pi_H} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)}$$

$$+\frac{(\Delta_1+\Delta_2-1) \quad \frac{d\tau_L^\pi}{-\tau_L^\pi} \quad \frac{v_L q_L}{{}_k vq} ''^\pi_L - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1+ \quad \frac{v_L q_L}{{}_k vq} ''^\pi_L}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{53}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^\pi_L} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)}$$

$$+\frac{(\Delta_1+\Delta_2-1) \quad -\frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1+ \quad \frac{v_H q_H}{{}_k vq} ''^\pi_H \quad + \frac{d\tau_H^\pi}{-\tau_H^\pi} \quad \frac{v_H q_H}{{}_k vq} ''^\pi_H}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{54}$$

F[illegible] (51)-(54) [illegible]

$$\begin{aligned}
\frac{dz_H^w}{d \quad {}^w_H}\frac{1}{z_H^w} &= \frac{''^w_H}{1- \quad {}^w_H} \quad \frac{(1- \quad)\frac{\delta_H l_H}{{}_k \delta l}''^w_H - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d \quad {}^w_H}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{{}_k \delta l}''^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d \quad {}^w_H}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{{}_k \delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d \quad {}^w_H}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{{}_k \delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{55}$$

$$\begin{aligned}
\frac{dz_H^w}{d \quad {}^w_L}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{{}_k \delta l}''^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d \quad {}^w_L}\frac{1}{z_L^w} &= \frac{''^w_L}{1- \quad {}^w_L} \quad \frac{(1- \quad)\frac{\delta_L l_L}{{}_k \delta l}''^w_L - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d \quad {}^w_L}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{{}_k \delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d \quad {}^w_L}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{{}_k \delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{56}$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_H^{\pi}}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_H^{\pi}}{-\tau_H^{\pi}}\frac{v_H q_H}{{}_m vq}{''}^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_H^{\pi}}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_H^{\pi}}{-\tau_H^{\pi}}\frac{v_H q_H}{{}_m vq}{''}^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^{\pi}}{d\ {}_H^{\pi}}\frac{1}{z_H^{\pi}} &= \frac{{''}^{\pi}_H}{1-\ {}_H^{\pi}}\ \frac{{''}^{\pi}_H\frac{v_H q_H}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^{\pi}}{d\ {}_H^{\pi}}\frac{1}{z_L^{\pi}} &= \frac{\frac{\varepsilon_H^{\pi}}{-\tau_H^{\pi}}\frac{v_H q_H}{{}_m vq}{''}^{\pi}_L}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_L^{\pi}}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_L^{\pi}}{-\tau_L^{\pi}}\frac{v_L q_L}{{}_m vq}{''}^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_L^{\pi}}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_L^{\pi}}{-\tau_L^{\pi}}\frac{v_L q_L}{{}_m vq}{''}^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^{\pi}}{d\ {}_L^{\pi}}\frac{1}{z_H^{\pi}} &= \frac{\frac{\varepsilon_L^{\pi}}{-\tau_L^{\pi}}\frac{v_L q_L}{{}_m vq}{''}^{\pi}_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^{\pi}}{d\ {}_L^{\pi}}\frac{1}{z_L^{\pi}} &= \frac{{''}^{\pi}_L}{1-\ {}_L^{\pi}}\ \frac{{''}^{\pi}_L\frac{v_L q_L}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{58}
$$

[illegible]

I [illegible], [illegible]. [illegible] L [illegible], [illegible], [illegible]

$$
\begin{aligned}
\max_{\tau_k^w,\tau_m^\pi} W = & \sum_k l_k \quad -c_w \ \frac{z_k^w}{M(\ )\,w_k} \quad + \sum_m q_m \quad -c_\pi \ \frac{z_m^\pi}{\frac{M\,\theta}{\theta}\quad {}_m}\\
& + {}_H l_H c_w' \ \frac{z_H^w}{M(\ )\,w_H} \ + {}_L l_L c_w' \ \frac{z_L^w}{M(\ )\,w_L} \ + v_H q_H c_\pi' \ \frac{z_H^\pi}{\frac{M\,\theta}{\theta}\quad {}_H} \ + v_L q_L c_\pi' \ \frac{z_L^\pi}{\frac{M\,\theta}{\theta}\quad {}_L} \ + R\\
& + \ (\ {}_k \ l)M(\ ) \ \frac{{}_H l_H}{{}_k\ l}\ {}_H^w w_H + \frac{{}_L l_L}{{}_k\ l}\ {}_L^w w_L + \frac{v_H q_H}{{}_m vq}\ {}_H^\pi\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}_L^\pi\ {}_L \ ;
\end{aligned}
$$

[illegible]. D [illegible]

$$
a = \frac{{}_H l_H}{{}_k\ l}\ {}_H^w w_H + \frac{{}_L l_L}{{}_k\ l}\ {}_L^w w_L \qquad b = \frac{v_H q_H}{{}_m vq}\ {}_H^\pi\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}_L^\pi\ {}_L :
$$

$$\frac{\partial L}{\partial \ {}^{w}_{H}} =$$

$$= \ _k \ l_k \ -\frac{c'_w}{M(\ )\, w_k}\frac{dz^w_k}{d \ {}^w_H} + \ _m \ q_m \ -\frac{c'_\pi}{\frac{M\ \theta}{\theta} \ _m}\frac{dz^\pi_m}{d \ {}^w_H}$$

$$+\frac{dz^w_H}{d \ {}^w_H}\frac{1}{M(\ )\, w_H} l_H c'_w \ \frac{z^w_H}{M(\ )\, w_H} + \ _H l_H c''_w \ \frac{z^w_H}{M(\ )\, w_H} \ \frac{1}{M(\ )\, w_H}\frac{dz^w_H}{d \ {}^w_H}$$

$$+\frac{dz^w_L}{d \ {}^w_H}\frac{1}{M(\ )\, w_L} l_L c'_w \ \frac{z^w_L}{M(\ )\, w_L} + \ _L l_L c''_w \ \frac{z^w_L}{M(\ )\, w_L} \ \frac{1}{M(\ )\, w_L}\frac{dz^w_L}{d \ {}^w_H}$$

$$+\frac{dz^\pi_H}{d \ {}^\pi_H}\frac{1}{\frac{M\ \theta}{\theta} \ _H} q_H c'_\pi \ \frac{z^\pi_H}{\frac{M\ \theta}{\theta} \ _H} + v_H q_H c''_\pi \ \frac{z^\pi_H}{\frac{M\ \theta}{\theta} \ _H} \ \frac{1}{\frac{M\ \theta}{\theta} \ _H}\frac{dz^\pi_H}{d \ {}^\pi_H}$$

$$+\frac{dz^\pi_L}{d \ {}^\pi_L}\frac{1}{\frac{M\ \theta}{\theta} \ _L} q_L c'_\pi \ \frac{z^\pi_L}{\frac{M\ \theta}{\theta} \ _L} + v_L q_L c''_\pi \ \frac{z^\pi_L}{\frac{M\ \theta}{\theta} \ _L} \ \frac{1}{\frac{M\ \theta}{\theta} \ _L}\frac{dz^\pi_L}{d \ {}^\pi_L}$$

$$+\left[\ _k \ \frac{l_k}{M(\ )\, w_k}\frac{dz^w_k}{d \ {}^w_H} \ M(\ ) + (\ _k \ l) M(\ ) \ \frac{ \ _m \ \frac{q_m}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz^\pi_m}{d\tau^w_H}}{ \ _k \ l} - \frac{(\ _m \ vq) \ _k \left(\frac{l_k}{M\ \theta\ \ w_k}\frac{dz^w_k}{d\tau^w_H}\right)}{(\ _k \ l)^2} \right] \ (a+b)$$

$$+ \ (\ _k \ l) M(\ )$$

$${}_H l_H\, c''_w \quad \frac{z^w_H}{M(\ )\, w_H} \quad \frac{1}{M(\ )\, w_H} \frac{dz^w_H}{d\ {}^w_H} + {}_L l_L\, c''_w \quad \frac{z^w_L}{M(\ )\, w}$$ [redacted]

$$+v_H q_H\, c''_\pi \quad \frac{z^\pi_H}{\frac{M\ \theta}{\theta}\quad {}_H} \quad \frac{1}{\frac{M\ \theta}{\theta}\quad {}_H} \frac{dz^\pi_H}{d\ {}^w_H} + v_L q_L\, c''_\pi \quad \frac{z^\pi_L}{\frac{M\ \theta}{\theta}\quad {}_L}$$ [redacted]

$$+\ (\quad {}_k\ \ l) M(\ ) \left[\frac{{}_k\left(\frac{l_k}{M\ \theta\ \ w_k} \frac{dz^w_k}{d\tau^w_H}\right.}{{}_k\ \ l} + \frac{M'(\ )}{M(\ )} \quad \frac{m}{\ }\right.$$ [redacted] $$\left.\frac{\frac{l_k}{M\ \theta\ \ w_k} \frac{dz^w_k}{d\tau^w_H}}{{}_k\ \ l} \right] (a+b) =$$

$$= {}_H l_H\, c''_w \quad \frac{z^w_H}{M(\ )\, w_H} \quad \frac{1}{M(\ )\, w_H} \frac{dz^w_H}{d\ {}^w_H} \frac{z^w_H}{z^w_H} \frac{M(\ )\,(}{c'_w(z^w_H =}$$ [redacted]

$$+\ {}_L l_L\, c''_w \quad \frac{z^w_L}{M(\ )\, w_L} \quad \frac{1}{M(\ )\, w_L} \frac{dz^w_L}{d\ {}^w_H} \frac{z^w_L}{z^w_L} \frac{M(\ )\,(1 - \ {}^w_L) w_L}{c'_w(z^w_L = M(\ )\, w_L)}$$

$$+v_H q_H\, c''_\pi \quad \frac{z^\pi_H}{\frac{M\ \theta}{\theta}\quad {}_H} \quad \frac{1}{\frac{M\ \theta}{\theta}\quad {}_H} \frac{dz^\pi_H}{d\ {}^w_H} \frac{z^\pi_H}{z^\pi_H} \frac{M(\ )}{\ } \frac{(1 - \ {}^\pi_H)\quad {}_H}{c'_\pi(z^\pi_H = \quad {}_H)}$$

$$+v_L q_L\, c''_\pi \quad \frac{z^\pi_L}{\frac{M\ \theta}{\theta}\quad {}_L} \quad \frac{1}{\frac{M\ \theta}{\theta}\quad {}_L} \frac{dz^\pi_L}{d\ {}^w_H} \frac{z^\pi_L}{z^\pi_L} \frac{M(\ )}{\ } \frac{(1 - \ {}^\pi_L)\quad {}_L}{c'_\pi\left(z^\pi_L = \frac{M\ \theta}{\theta}\quad {}_L\right.}$$

$$+\ (\quad {}_k\ \ l) M(\ ) \left[\quad {}_k\left(\frac{l_k}{M\ \theta\ \ u\mathrm{wH}}\right.\right.$$

$$= \ {}_{H} l_{H} \frac{1}{''^{w}_{H}} \frac{dz^{w}_{H}}{d \ ^{w}_{H}} \frac{1}{z^{w}_{H}}$$

${}^{\pi}_{H}, \quad {}^{\pi}_{L}$

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad)^{1 - \quad {}^{w}_{L}}$$

F

$$= 1 - \frac{\left[\begin{aligned} &\left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{HL} \quad {}^{w}_{H} \\ &+ \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{LL} \quad {}^{w}_{L} \\ &+ \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{HL} \quad {}^{\pi}_{H} \\ &+ \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{LL} \quad {}^{\pi}_{L} \end{aligned}\right]}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H}\ {}_H + \frac{-\tau^\pi_L}{\varepsilon^\pi_L}\ {}_L} \; ;$$

$$= \left(1 - \frac{E_{(s)} w_{Hs}\ {}^{w}_{H} + E_{(s)} w_{Ls}\ {}^{w}_{L} + E_{(s)}\ {}_{sH}\ {}^{\pi}_{H} + E_{(s)}\ {}_{sL}\ {}^{\pi}_{L}}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H}\ {}_H + \frac{-\tau^\pi_L}{\varepsilon^\pi_L}\ {}_L}\right)^{-} ; \qquad (64)$$

s $\left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \ldots\right.$

c'

$$'' = \frac{1}{}\frac{c''}{c'} = \frac{1}{-1}.$$

, A, :

.

2. $c = A(\ ^{\gamma} + \ ^{\beta})$ ≥ 3, $<$, $c' = A(\ ^{\gamma-} + \ ^{\beta-})$,

$$c'' = A(\ (\ -1)\ ^{\gamma-2} + \ (\ -1)\ ^{\beta-2}) > 0;$$

$$'' = \frac{\ ^{\gamma-2} + \ ^{\beta-2}}{(\ -1)\ ^{\gamma-2} + (\ -1)\ ^{\beta-2}}.$$

A , , .

$$\frac{@''}{@} = -\ ^{\beta}\ ^{\gamma-}\ (\ -\ )^2 < 0.$$

N ≥ 3 $=$ -1. . F , < 1 $= 3$ $= 2$, > 1 $= 2$ $= 1$.

$\frac{-\tau_H^w}{\varepsilon_H^w} w_H < \frac{-\tau_L^w}{\varepsilon_L^w} w_L$, (66) -. H $(1 - \ _H^w) w_H < (1 - \ _L^w) w_L$ $\ _H^w > \ _L^w$ (66) $\ _H^w w_H > \ _L^w w_L$. N $(E_{(m)}\ _{Hm}\ _m^{\pi} - E_{(m)}\ _{Lm}\ _m^{\pi})$. - $\ _{HH} > \ _{LH}$ $\ _{HL} > \ _{LL}$ $(E_{(m)}\ _{Hm}\ _m^{\pi} - E_{(m)}\ _{Lm}\ _m^{\pi} > 0)$. H (66) . $(1 - \ _H^w) w_H > (1 - \ _L^w) w_L$, $\ _H > \ _L$, $''^w_H \leq ''^w_L$. □

Proof of Proposition 11.

(65). (60)-(63) . , . D

(60) (61)

$$(\Delta_1+\Delta_2-1)\quad(1-\ )\frac{1-\ ^{w}}{''_{w}}w+\quad(w\ \ ^{w}+\quad\ ^{\pi}-(1-\quad)\bar{R})\quad=$$

$$=(1-\quad)\left[(1-\ \ ^{w})w+\quad(w\ \ ^{w\,''w}+\quad\ ^{\pi\,''w}-(1-\quad)\bar{R}^{''w})\right]$$

$$-$$

(68), (69), P 11.

P 12

$(1 -\ ), (1 -\ ) = \uparrow$,

$^{\pi} = {}^{w} \downarrow$. □