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## Price-Directed Consumer Search

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## Abstract

This paper presents an oligopoly model in which consumers conduct sequential costly search for a desired product. In contrast to the specification of random or pre-determined search order in most current studies, I allow the order of consumer search to be endogenously determined. In the model, consumers observe product prices before searching among firms. Thus, price additionally affects profit by influencing the order in which consumers search. I find that the pattern of equilibrium price distribution depends on the size of search cost. In particular, for a medium search cost, a mixture price distribution prevails: firms randomly play two separate price distributions with a gap in between. Firms either price high, following a high price distribution, or price low, following a low price distribution, but always avoid prices at the intermediate interval. For low or high search costs, equilibrium price distribution is continuous with positive density on the entire support. Comparative statics show that equilibrium price is non-monotonic in search cost and firm profit can be higher when consumers have lower search costs.

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# 1 Introduction

An extensive literature has examined how consumers' costly sequential for information affects market performance. These models typically assume that consumers search randomly among firms. Stahl (1989) analyzes a model where firms sell homogenous goods, and consumers with positive search costs visit firms in a random order and buy from the first firm that offers a price below their reservation price. Wolinsky (1986) and Anderson and Renault (1999) consider differentiated product markets in which consumers incur costs to obtain price as well as product information. Consumers examine each product with an equal probability and purchase if a product yields a sufficient match to their preference. In these models

instance, with price search engines, consumers may easily obtain a list of prices of products by different sellers. However, the costs of time and effort for consumers to find out if a product matches their preferences remain significant. With observable prices, an interesting twist occurs. Prices, in addition to entering into firms' profit function directly, it may also influence the order in which consumers search, thus, indirectly affect firms' profits. By charging a relatively low price, a firm may become more "prominent" and increase sales. Surprisingly, the latter strategic consideration has not been explored in the literature. This paper attempts to fill in this gap.



Equilibrium analysis is conducted in section 3. Section 4 shows comparative statics on search cost. Section 5 concludes.

## 2 The model

There is a continuum of consumers, each with a "need." A consumer derives utility  $V$  if her need is met and 0 otherwise.  $N$  firms exist in the market, each of which carries a product that meets a consumer's need with a certain probability.<sup>1</sup> The probability is assumed independent and identical for all firms and is denoted as  $\alpha \in [0; 1]$ . The marginal cost is assumed constant and normalized to zero.

There are two types of consumers: informed and uninformed. In particular,  $\alpha$  of consumers are informed knowing if products meet their needs (without search). The remaining  $1 - \alpha$  of consumers are uninformed. An uninformed consumer initially does not know if a product meets her need but can learn it by incurring a search cost  $s$ . A product meets a consumer's need with probability  $\alpha$ .

Since an informed consumer observes all prices and knows if a product matches her preference she will purchase from the matched firm (if any) with the lowest price in the market. An uninformed consumer has to decide a search strategy to maximize her expected payoff. Given that a firm charges price  $p$ ; an uninformed consumer will search the firm only if the expected payoff is non-negative, that is,

$$(V - p) - s \geq 0$$

or

$$p \leq V - s:$$

Define

$$r = V - s: \tag{1}$$

We shall call  $r$  as reservation price. Thus, an uninformed consumer will only search a firm if the firm charges a price that is equal to or below the reservation price, that is,  $p \leq r$ : Next, we consider the order by which uninformed consumers search among firms charging prices below the reservation price. Given that each firm has a same probability to match a consumer's preference, uninformed consumers get





$$G_i = \frac{1}{1} \frac{1}{(1 - \frac{V}{p})^{N-1}} \frac{V}{p} \frac{1}{1} \quad (4)$$

$$= \frac{1}{1} \frac{V}{r} \frac{1}{(1 - \frac{V}{r})^{N-1}} \frac{1}{1} \quad (5)$$

and  $r$  is defined as in (1).

Proof. We first verify that  $G(p)$  is a c.d.f. Since  $G(V) = 1$ ;  $G(1 - \frac{V}{p})^{N-1} V = 0$ ,  $G(V - \epsilon) = G(\frac{V - \epsilon}{r}) = 1 - \frac{1 - \frac{V - \epsilon}{r}}{(1 - \frac{V - \epsilon}{r})^{N-1}} = 1 - \frac{1 - \frac{V - \epsilon}{r}}{(1 - \frac{V - \epsilon}{r})^{N-1}}$  and  $G(p)$  weakly increases in  $p$ ; it follows that  $G(p)$  is a continuous c.d.f.

We next show that each firm is optimizing following  $G(p)$ , given that other firms choose prices according to  $G(p)$  and uninformed consumers' reservation price is  $r$ : Note that a firm can only sell to an (informed and uninformed) consumer if its price is the lowest among the matched firms. In addition, selling to a uninformed consumers requires the price is lower than  $r$  to induce a search. The expected profit when a firm chooses  $p$  is:

(i) If  $p = V$ ;

$$\pi = V (1 - \frac{V}{p})^{N-1} \quad (6)$$

because the firm only sells to the informed consumers who find the firm is the only match.

(ii) If  $p = r$  a firm sell to matched consumers (informed or uninformed) who find that the firm has the lowest price. That is, firms with lower prices do not match. Thus, the profit is

$$\begin{aligned} &= r [(1 - \frac{V}{r})^{N-1} + \frac{N-1}{1} (1 - \frac{V}{r})^{N-2} + \frac{N-1}{2} (1 - \frac{V}{r})^{N-3} + \dots \\ &+ \dots + \frac{N-1}{i} (1 - \frac{V}{r})^{N-1-i} + \dots + \frac{N-1}{N-1} (1 - \frac{V}{r})^{N-1}] \\ &= r \end{aligned}$$

(iii) if  $p \geq G_h(p)$  or  $p > r$ ;

$$\begin{aligned}
 &= p \sum_{i=0}^{N-1} \binom{N-1}{i} (1 - G_h(p))^{N-1-i} (1 - G_h(p))^i \\
 &= p [ (1 - G_h(p)) + 1 - G_h(p) ]^{N-1}
 \end{aligned} \tag{8}$$

because the firm only sells to the informed consumers who find its price is lowest among the matched firms.

(iv) If  $p \geq G_l(p)$  or  $p < r$ ;

$$\begin{aligned}
 &= p \sum_{i=0}^{N-1} \binom{N-1}{i} (1 - G_l(p))^{N-1-i} [ f + (1 - G_l(p)) (1 - G_l(p)) ]^i \\
 &= p f [ f + (1 - G_l(p)) (1 - G_l(p)) ] + 1 - G_l(p)
 \end{aligned} \tag{9}$$

because the firm can sell to both informed and uninformed consumers when the its price is lowest among the matched firms.

Equal profit from (i) and (ii) yield (5), Equal profit from (i) and (iii) yield (3). Equal profit from (i) and (iv) yield (4). Therefore, the firm optimizes choosing prices according to  $G(p)$ :

Finally, note that when  $h > s > (1 - G_l(p))^{N-1} V > s > (1 - G_l(p))^{N-1} V$

probability and 1

Substituting (3), (4) and (5) into (2), the price distribution can be rewritten as

$$G(p) = \int_0^{\infty} \left( 1 - \frac{1 - \frac{v}{i^p v}}{N-1} \right) \cdot \mathbb{1}_{\{v \leq p < (1 - \frac{v}{i^p v})^{N-1} v\}} dv$$

As shown in Figure 1a, the equilibrium price distribution function stays constant over some intermediate prices. This implies zero probability placed on these prices.

We next discuss the case where  $s$  is small.

Proposition 2 When  $s < (1 - \frac{1}{N}) V$  there exists a symmetric equilibrium, in which each firm prices according to mixed strategy

$$F(p) = 1 - \frac{1}{\frac{r}{p} - 1} \left( \frac{r}{p} \right)^{\frac{1}{N-1}} \quad \text{if } r \leq p \leq (1 - \frac{1}{N}) r; \quad (12)$$

Proof. Given  $F(p)$ , the uninformed consumers search optimally. To show that the proposed is an equilibrium, we thus only need to show that given  $r$  and other firms choose  $F(p)$ , each firm optimizes choosing any  $p \in (1 - \frac{1}{N}) r; r$ : For any such price, the firm's expected profit is

$$\begin{aligned} & p \sum_{i=0}^{N-1} \binom{N-1}{i} (1 - F(p))^{N-1-i} (1 - G_1(p))^i \\ &= p \left[ (1 - F(p)) + 1 - \frac{1}{\frac{r}{p} - 1} \left( \frac{r}{p} \right)^{\frac{1}{N-1}} \right]^{N-1} \\ &= p \left[ 1 - \frac{1}{\frac{r}{p} - 1} \left( \frac{r}{p} \right)^{\frac{1}{N-1}} \right]^{N-1} \\ &= r \left( 1 - \frac{1}{\frac{r}{p} - 1} \left( \frac{r}{p} \right)^{\frac{1}{N-1}} \right)^{N-1} \end{aligned}$$

Note that  $p > r$  would lead to zero sale and any





## 4 Comparative Statics on Search Cost

Our previous analysis shows how the equilibrium price strategies depend on if the



the equilibrium price stochastically; (ii) increases firms' profits; and (iii) increases total welfare.

When  $s$  is small, firms choose a price strategy with all prices below the uninformed consumers reservation price,  $r$ . By (1), as  $s$  decreases,  $r$  increases. This allows firms charge higher prices resulting in a higher profit. Note that there is no welfare loss as long as consumers with at least one match will find her matched products and purchase. This is true in this equilibrium. Moreover, there is a reduction in search cost. Therefore, total welfare increases. Note that the change of consumer welfare is not clear. On one hand, the lower search cost benefits consumers. On the other, equilibrium prices increase which lowers consumer welfare.

When  $s$  is medium, by (10),  $G(p)$  is weakly increasing in  $s$ . Moreover, the limits of upper bound,  $\frac{V-s}{N}$  and lower bound,  $V - s$ , increase in as  $s$  decreases. Therefore, the price distribution  $G(p)$  stochastically increases, thus, the equilibrium prices are stochastically lower as  $s$  decreases. Figure 2b illustrates the change of  $G(p)$  for a decrease in search cost.

Proposition 5 When  $s$  is medium, that is  $\frac{1}{N} < s < \frac{V}{N}$ , a decrease in  $s$  (i) decreases the equilibrium prices stochastically; (ii) increases consumer surplus; (iii) does not affect firms' profits; and (iv) increases total welfare.

When  $s$  is medium, firms adopt a clustered price distribution: randomly playing a high price distribution with prices above  $r$  and a low price distribution with price below  $r$ . As before,  $r$  increases as



Several points that are worth to notice. First, equilibrium prices are non-monotonic in search cost and reach their minimum at some intermediate search costs. Second, since the welfare of informed consumers is positively correlated with the expected minimum market price it follows that a decrease in uninformed consumers' search cost may exert positive, negative or none externality to informed consumers. Third, firms earn more profit with a lower consumer search cost. In fact, maximum profit is obtained when uninformed consumers have zero search cost. Equivalently, profit maximizes when consumers are all informed.

## 5 Conclusion

In many situations, consumers need to search costly for goods or services to meet their needs. The search often is conducted in a non-random order. This paper presents a model with endogenous consumer search order. The rapid development of Internet allows consumers to have easy access of price information before searching for the desired product. In the model, prices affect firms' profits directly and indirectly via influencing the order in which consumers search. Firms charging lower prices appear on the top of the consumers' search order, and thus, make more sales. We show that the pattern of equilibrium price distribution depends on the size of search cost. In particular, mixture price distribution occurs when the search cost is medium while standard continuous price distribution appears when the search costs are high or low. We find that a decrease in search cost may increase, decrease or have no impact on equilibrium prices depending on the initial size of search cost; and market price minimizes at some medium size of search cost. Moreover, a decrease in search cost weakly increases firms' profits.

For future research, it would be interesting to compare a price-directed search model as in this paper to a random search model where consumers search costly for both price and product information. In the comparison, one could examine how price patterns and the impact of search costs on welfare differ in these models.

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