

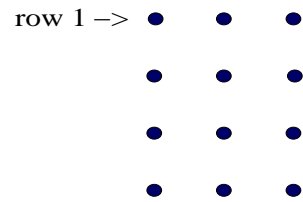
Program in Applied Mathematics  
PROBABILITY AND STATISTICS PRELIMINARY EXAMINATION  
August 2008

**Notice:** Do four of the following five problems. Place an X on the line opposite the number of the problem that you are NOT submitting for grading. Please do not write your name anywhere on this exam.

1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_

- (b) Find the best (most powerful) test of size  $\alpha$  of  $\mu = 0$  versus  $\mu = \mu_1$ , based on observing  $W_1, W_2, W_3$ , and  $W_4$ .
- (c) Is there a uniformly most powerful test of  $\mu = 0$  versus  $\mu > 0$  based on  $W_1, W_2$ , and  $W_3$ ? Explain.

4. A subset of 4 dots is selected from a 4 by 3 rectangular array of dots depicted below.



- (a) What is the probability that no dot from the first row is selected?
  - (b) Find the expected number of rows with no selected dots.
  - (c) Find the variance of the number of rows with no selected dots.
5. Let  $N(t)$  be a Poisson counting process with rate  $\lambda$ . Let  $G(t)$  be a Gamma process, with parameters  $\alpha$  and  $\beta$ , which is defined as follows.  $G(t)$  has stationary independent increments, with  $G(0) = 0$  and  $G(s + t) - G(s) \sim (\alpha t, \beta)$ , where  $X \sim (\alpha T, \beta)$  means  $f_X(t) = \frac{\alpha^\beta}{\Gamma(\beta)} t^{\beta-1} e^{-\alpha t}$ .