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Interactions of dispersive shock waves (DSWs) and rarefaction waves (RWs) associated with the Korteweg–de Vries equation are shown to exhibit multiphase dynamics and isolated solitons. There are six canonical cases: one is the interaction of two DSWs that exhibit a transient two-phase solution but evolve to a single-phase DSW for large time; two tend to a DSW with either a small amplitude wave train or a finite number of solitons, which can be determined analytically; two tend to a RW with either a small wave train or a finite number of solitons; finally, one tends to a pure RW.

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Shock waves in processes dominated by weak dispersion and nonlinearity have been experimentally observed in plasmas $[1]$ $[1]$ $[1]$, water waves $[2]$ $[2]$ $[2]$, and more recently in Bose-Einstein condensates $[3,4]$ $[3,4]$ $[3,4]$ $[3,4]$ and nonlinear optics $[5]$ $[5]$ $[5]$; these dispersive shock waves (DSWs) have yielded novel dynamics and in-

2) using the "steplike"

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to be 0, 1 and $0 \lt \sqrt{1}$ (by using a scaling symmetry and Galilean invariance). The case of a well (e.g., $h_0 = 2 = 0$ \ge *h*₁) and a box (e.g., $h_0 = 2 = 0$ \le *h*₁) with vanishing boundaries was studied in $[7]$ $[7]$ $[7]$, where the asymptotic solution was constructed analytically.

This paper is organized as follows. We first discuss

$$
_{0}(.)=\begin{cases} 0, < 0\\ 0, < < 0\\ 1, < < 0\\ 2, > , \end{cases} \tag{3}
$$

where $_0$, $_1$, and $_2$ are distinct, real, and non-negative. This gives six canonical cases, which we denote as

$$
I(- \t): h_0 > h_1 > h_2, \tI(\t): h_0 > h_2 > h_1,
$$

\n
$$
III(\t): h_1 > h_0 > h_2, \tIV(\t): h_2 > h_0 > h_1,
$$

\n
$$
2 > h_0, \tVI(\t): h_2 > h_1 > h_0,
$$

Although the (initial) shock front speed is different for DSWs and VSWs $(2 \t_0/3$ and \t_0

In case IV $\,$, a small DSW forms on the left and a large RW forms on the right see Fig. 6 a . As in case II , the front of the DSW interacts with the trailing edge of the RW and decreases the DSW's amplitude and speed. Unlike case II $\qquad \quad$, the front of the DSW does not

6 b

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