ey^{ϕ} duaft of INgeria lectures, May 1991 f

Wavelets, Multiresolution Analysis and Fast Numerical Algorithms

Bey n

 $\mathcal{F}=\mathbb{A}$ Multipole \mathcal{F} order p j $$ i j q_iq_j | ⁱ

 \mathcal{F} ese e ods ye e edste cesto ed ci \mathbb{I} partial equation of \mathbf{e} sparse linear system for the cost of a indicate cost of e of α in α ces. If $n \geqslant e$ dof nederlance or neee eneper on $p \geqslant e$ be ep $r = \tan \theta$ on of ede ean ee as en ape od c on

 \mathbf{P} e^l \mathbf{P} y e \mathbf{P}

II.1 Multiresolution analysis.

 $\begin{array}{lllllllll} \textbf{I} & \textbf{e} & \textbf{e}$ y Meyer, and Mallat \Box cpe extended

If $e \cdot e \cdot e$ nen e of $x \cdot e \cdot e$ en o ∞ of $\int_0^\infty e^{e^{-x}} e^{-x}$ o e e $ne \neq e$ n $ne \neq d$ of e e en e

$$
\bm{V_n} \subset \quad \subset \bm{V} \ \subset \bm{V} \ \subset \bm{V} \quad \ \bm{V} \ \subset \bm{L} \ \ \bm{R}^d
$$

nn ec e z ona ea p ce V a n e d en aon

II.2 The Haar basis

o only eeony epecte e pecte $\frac{1}{2}$ on $\frac{1}{2}$ fyn $\frac{1}{2}$ Den on Condition θ as the Haar basis computing in the Haar basis θ as the Haar basis of θ of e loops that all describe in the algorithms of entropype \mathbf{e} and \mathbf{p} fon ecepe en on f d = en \blacksquare = j = j = j = τ ∈**Z** afored y ed on nd na on of an^{ta} efine on

$$
\begin{array}{c}\n\mathbf{g} \\
\hline\n\mathbf{g} \\
\hline\n\mathbf{
$$

In $\clubsuit c \clubsuit$, where is the characteristic function of the interval (1). For eac j j;k = j= (2 ^j −), ∈ Z is the basis of V^j and j;k = j= (2 ^j −), ∈ Z is the basis of W^j

 \mathcal{F} e decomposition of function into the Haar basis is an order N pocedure. en $N = \n\begin{bmatrix} \n\bullet & \bullet \\
\bullet & \bullet\n\end{bmatrix}$ of \bullet function, \bullet y for \bullet p c y e of of \bullet expected by \bullet of x ed e least f on ne and entremely near \mathbf{r}

$$
\mathbf{k} \leftarrow \mathbf{h} = \begin{bmatrix} \mathbf{Z} & -n & \mathbf{k} + \\ & -n & \mathbf{k} \end{bmatrix} \mathbf{f} \quad d
$$

e o $n \blacktriangleright$ coe cen \blacktriangleright

$$
d_{\mathbf{k}}^{\mathbf{j}+} = \frac{1}{\sqrt{2}} d_{\mathbf{k}}^{\mathbf{j}+} d_{\mathbf{k}}^{\mathbf{k}+} = d_{\mathbf{k}}^{\mathbf{j}+}
$$

nd e $\int_{\mathbb{R}}$

$$
\begin{array}{ccc}\n\mathbf{j}^{+} & -\frac{1}{\sqrt{ }} & \mathbf{j}^{+} \\
\mathbf{k} & \mathbf{k}^{+} & \mathbf{k}^{+} \\
\end{array}
$$

for $j = n - \text{nd}$ \bullet j \bullet \bullet \bullet easy to see that evaluating the whole \clubsuit of coe c en $\clubsuit d_{\mathbf{k}}^{\mathbf{j}}$ k j $\lim_{k \to \infty}$ in eq e.⁹ $N-$ dd on a nd $N-$ p c on a noden, on are e on \overline{a} 96, \overline{m} 89.84 -14.4 Tdf5.7564 0 Td 1f^tdder $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ o e and d form. The second basis is defined y the set of the nd of λ functions supported on squares: j;k j;k⁰ y), j;k j;k⁰ y), and j;k j;k⁰ y), where λ egg ce acfric on of ene ind $j_{jk} = j = j - j$.
The employment in operator in the terminology of the termino ecoe ce By considering an integral operator operator of \mathbb{R} and \mathbb{R} operator of \mathbb{R} \mathbf{z}

$$
f \qquad - \qquad y \, f \, y \, dy
$$

and expanding its ernel in ordinal \blacksquare in \blacksquare and to C deterministic that for C deterministic multiple in \blacksquare Zy^{gm} and and p_{re} do_nderen operation of the decay of entries as function of the decay of entries as function of the experimental operators as function of the decay of entries as function of the decay of entries as fu denote diffond a factor is the diagonal in the original is faster in the original in the original α enel. These classes operators are given y net only and only energy e to be difference been the diagonal. For example, ernels and y of C determinal y and ope 0.22 $4y$ ee2 e2

$$
\begin{array}{c|c|c|c|c|c|c|c} & y & | & \leq & \hline & & & \\ \mid & & & & & & & \\ \mid & & & & & & & & \\ \mid & & & & & & & & & \\ \mid & & & & & & & & & \\ \mid & & & & & & & & & \\ \mid & & & & & & & & & & \\ \mid & & & & & & & & & & & \\ \mid & & & & & & & & & & & \\ \mid & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & & & & & & & & & & & & & \\ \mid & &
$$

fo \clubsuit e $M \geq 1$ Le $M = \begin{array}{ccc} n & \text{nd} & \text{con} & \text{ad} \\ \textbf{Z} & \textbf{Z} & \end{array}$ j $\mathbf{y} = \begin{bmatrix} 1 & y & z \\ z & z \end{bmatrix}$ $\mathbf{y} = \begin{bmatrix} 1 & y & z \\ y & z & z \end{bmatrix}$ $\mathbf{y} = \begin{bmatrix} 1 & y & z \\ y & z & z \end{bmatrix}$ e e e assume that the et as unce et een $| - | \ge 1$ note Z

$$
j_{ik} \quad d_{l} = 0
$$

e e

|264ψTj /R33ψiy052ψTf 9r.14)x s1.9552 Tf

 $r_{\rm e}$ and $r_{\rm e}$ is in the decay and computing in the Hampshire $r_{\rm e}$ T_0 e factor y_a ancess y o use basis functions with several and \mathbb{R} oen a_k fe n and oen a eeponsie for any practical algorithms, i.e. controlling the constants in the complexity estimated \mathbb{R}^n and \mathbb{R}^n algorithms. The fast algorithms are fast algorithms.

II.3 Orthonormal bases of compactly supported wavelets

 T_{eq} equestion of the existence of ultimate \bullet on α yields \bullet or α of α on α of α Condion e and e as edimencial construction of the order bases of e Fene \mathbf{z} m^o \mathbf{z} fine on \mathbf{z} y o e^{rg} \mathbf{z} , nd Meyer \mathbf{z} (e consider only compactly supported electric moments and \mathbf{n}^* or entropy constructed yield yield \mathbf{p} and \mathbf{p} \Box , foonless of Y. Meyer, nd Mallat of $\circ \not \Rightarrow$ of ee en d fooecoces of e^e \rightarrow \rightarrow \rightarrow e \rightarrow c \rightarrow c \rightarrow coce of ee \bullet y only considering for dimensions d \geq 2, e \bullet \bullet \bullet constructed for $\alpha \bullet$ for $d =$

Let \mathcal{A} con \mathcal{A} der the ultima analysis \mathcal{A} and \mathcal{A} \mathcal{A} on a \mathcal{A}

econd e o o^ron y of { − }k z p e, $k \stackrel{\sim}{\sim}$ $-$ d $\left| \cdot \right| \cdot \left| \cdot \right|$ e^{ik} $d \cdot \left| \cdot \right|$

 nd e efo e

$$
\begin{array}{ccc}\n\mathbf{z} & \mathbf{x} \\
\mathbf{z} & \mathbf{z}\n\end{array}
$$

 nd

$$
\sum_{i=1}^{k} |x_i|^2 \leq \sum_{i=1}^{k} |x_i|^2
$$

 $\mathbf{e} \circ \mathbf{n}$

 $\bar{}$. $\frac{1}{2}$

$$
\begin{array}{ccccccccc}\n\text{and} & c & o & \text{ao} & \text{see} & \text{the} & \text{one} & \text{one} & \text{the} &
$$

Lemma II.1 Any trigonometric polynomial solution \rightarrow of (2.26) is of the form

$$
r^{\prime} \quad \leftarrow \quad \frac{\mathsf{h}}{2} \qquad \mathrm{e}^{\mathrm{i}} \qquad \mathrm{e}^{\mathrm{i}} \qquad \mathrm{e}^{\mathrm{i}}
$$

where $M \geq$ is the number of vanishing moments, and where is a polynomial, such that

| = P(sin ¹ 2 ξ sin ^M ¹ 2 ξ 1 2 cos ξ (2.33) P y = k XM k=0 M − y k (2.34)

where

and is an odd polynomial, such that

 $\overline{}$

$$
\leq P \, y \qquad y^{\mathsf{M}} \quad \frac{1}{2} - d \qquad \qquad \qquad \mathbf{P}^{\mathsf{M}} \qquad \math
$$

 \boldsymbol{f}

e e j $\frac{d}{dx}$ nd $\frac{d}{dx}$ yeed apend caped ence equence eperiodic in j Coputing if nd \rightarrow \rightarrow ed y epy d \rightarrow e e $\{ k \} \longrightarrow \{ k \} \longrightarrow \{ k \} \longrightarrow \{ k \}$ \searrow \searrow \searrow $\left\{ d_{\mathbf{k}}\right\} \qquad \quad \left\{ \text{${\rm \mathit{\overset{\circ}{d}}}$} \right.$

ee $\mathsf{V}_\mathsf{j}^\mathsf{M}$ and e as the $\mathsf{W}_\mathsf{j}^\mathsf{M}$; as the orthogonal complement $\mathsf{V}_\mathsf{j}^\mathsf{M}$; j n V^{M;} j $\mathsf{V}_{\mathsf{j}}^{\mathsf{M};\mathbb{}}$ $-\mathsf{V}_{\mathsf{j}}^{\mathsf{M};\mathbb{}}$ ^{M;} ''' W^{M;}

 \mathcal{F} e ap ce **W^{M;}** ap nned y e o ono**rginal basis**

$$
\{ \begin{array}{ccccccccccc} i & 1 & y & i & 1 & y & b & t & b & M \end{array}
$$

 $\mathbf{e} \in \{ \text{ m/m }^{\mathbf{m} \mathbf{M}} \neq \mathbf{n} \}$ ono \mathbf{A} afo $\mathbf{V}^{\mathbf{M}}$ \mathbf{e} ee en on $\mathbf{e}^{\mathbf{m}}$ ex M ee en a na m^oo en a e ao o^rono e po yno a ⁱyⁱk, l $\frac{M-1}{2}$

 \mathcal{F} e pe w^{M} ; ap nned y dilations and translations of the basis \mathcal{F} W^{M;} nd e **a**nd α is a consistent of the part in one in the low-order polynomials α $\mathbf{i} y^{\mathsf{T}}$ \mathbf{M} \mathbf{M} \mathbf{M} \mathbf{M}

 \mathbf{V} e noe e oden, zonal ultimature bases require M different comn onto one-dimensional basis functions of $M \neq en$ is the num ergod model of anishing momentum or \mathbf{a} en \clubsuit On eoe nd e oden \clubsuit on \clubsuit \clubsuit o ned y \clubsuit n^s copcy apported electrical equipment on \bullet combinations \bullet be \bullet three substitutions which simplifies the con- \bullet c on of enon- \bullet nd d for \bullet see \circ on

II.5 A remark on computing in the wavelet bases

. In y e note once the \bullet een comparison between \bullet and \bullet the filter \bullet functions and and eefoe e_{μ} exponency as interesting observations. on in properly constructed \mathbb{r}_0 in the functions and are never computed. De o e ec \clubsuit e de n on of e e \clubsuit e are manipulations are performed bases, and manipulations are perfo eq de oes indeen filters and even if the q in \bullet as the ed md As ne peese computer and $A \bullet n$ and $A \bullet n$ and $A \bullet n$ are moments of the scaling function. T e e p exponsions en en a

$$
\mathcal{M}^{\mathsf{m}} = \begin{bmatrix} \mathbf{Z} & \mathsf{m} \\ \mathsf{m} & \mathsf{d} \end{bmatrix} \quad \mathcal{M} = \begin{bmatrix} M - \mathsf{m} \\ \mathsf{m} \end{bmatrix}
$$

n e **a** of e e coe c en $\mathbf{a} \{ k \}^{k}$ L y e fond an^f fo fo.

$$
\mathbf{y} = \begin{bmatrix} \mathbf{y} & \mathbf{y} & \mathbf{y} \\ \mathbf{y} & \mathbf{y} & \mathbf{y} \end{bmatrix} \mathbf{y}
$$

e e

$$
r^{\prime} \quad \stackrel{\longrightarrow}{\iff} \quad \frac{\ast \times}{\ast} \quad \frac{\ast e^{ik}}{k}
$$

 T e_s o en $\mathbf{A}^{\mathcal{M}^{\mathsf{m}}}$ e o ned n e c y n e desired accuracy) y ec \rightarrow e y \uparrow energies of eco \rightarrow $\{M_{\uparrow}^{m}\}_{m}^{m}$ m for r =

$$
\mathcal{M}^m_{r+} = \frac{jx^m}{j} \stackrel{p^m}{\rightarrow} \quad \text{if } \mathcal{M}^m_r \quad j\mathcal{M}^j
$$

 \rightarrow \mathbf{n}^*

$$
\mathcal{M}^m = m \frac{1}{2} \bigg| \bigg\|_{k}^{k} \sum_{k=1}^{m} m \bigg\|_{p}^{2} = M -
$$

ceco $\{\mathcal{M}_r^m\}_{m=1}^m$ eperta M oen and the podcn real and \neq e on con elles pdy. No ce enee coped efine on \rightarrow f

e non 3 nd d nd 3 nd d fo 3

III.1 The Non-Standard Form

Le e n ope o

 $L R \rightarrow L R$ e e ne y Denning projection operators on the subspace V_j j $\in \mathbb{Z}$

 P_j L R \rightarrow V_i

 \clubsuit

$$
P_{\mathbf{j}}f = \sum_{\mathbf{k}} \langle f | \mathbf{j}_{\mathbf{k}} \rangle \mathbf{j}_{\mathbf{k}}
$$

nd e p nd n^{ot} n "e e copic" series, e o n $$ j j P j j \longrightarrow

j Z

$$
\epsilon
$$

e e

$$
\mathbf{j} \cdot -P_{\mathbf{j}} - P_{\mathbf{j}}
$$

a epoeconope o on ea pece Wj fee a ecoapage e and e $n \geqslant e$ d of g e e

$$
\begin{array}{ccc}\n\mathbf{x} & \mathbf{j} & \mathbf{j} & \mathbf{j} & P_{\mathbf{j}} & P_{\mathbf{j}} & \mathbf{j} & P_{\mathbf{n}} & P_{\mathbf{n}} \\
\mathbf{y} & \mathbf{y} & \mathbf{y} & \mathbf{y} & P_{\mathbf{j}} & \mathbf{j} & P_{\mathbf{n}} & P_{\mathbf{n}}\n\end{array}
$$

nd f $e \cdot \mathbf{r} = \mathbf{r}$ e ne $\mathbf{r} = e$ e en

$$
\begin{array}{cccccccc}\n\mathbf{x} & & & & \\
\hline & & & & & \\
\mathbf{j} & & \mathbf{j} & & \mathbf{j} & P_{\mathbf{j}} & P_{\mathbf{j}} & & \mathbf{j} & P_{\mathbf{n}} & P_{\mathbf{n}} & \\
\end{array}
$$

ee ∼ $-P$ P is deez on of eope of on e net x e p n sions \mathbf{a} external decompose the operator into sum on \mathbf{a} from \mathbf{a} of contributions \mathbf{a} deren $x e$ \mathcal{F} enon-standard form is representation \mathcal{F} e. \mathcal{F} of eoperator as chain \mathcal{F} of $pe \geqslant$ $-\{A_j \ B_j \ A_j\}$ jz

c \mathfrak{m}^* on the subspace \mathcal{N}_j and \mathcal{W}_j

 $A_{\mathbf{i}} \quad \mathbf{W}_{\mathbf{i}} \rightarrow \mathbf{W}_{\mathbf{i}}$ B_i $V_i \rightarrow W_i$

 $\overline{\mathcal{M}}_j \longrightarrow V_j$ e e e operators ${A_j \t B_j \t A_j}$ j z e defined as A_j = j j B_j = j P_j and \cdot j $-P$ j j \mathcal{F} eopeo $\blacktriangleleft \{A_{\mathbf{j}} \ B_{\mathbf{j}} \ \ , \ \ {\mathbf{j}}_{\mathbf{j}} \ \ {\mathbf{z}} \ \ {\mathbf{d}} \qquad \quad \mathrm{ec} \ \ \ {\mathbf{z}} \ \ \mathrm{eden} \ \ \mathrm{on} \qquad \quad \ \mathrm{e} \ \ {\mathbf{e}} \quad \mathrm{on}$

$$
\mathbf{j} = \begin{array}{cc} A_{\mathbf{j}+} & B_{\mathbf{j}+} \\ B_{\mathbf{j}+} & \mathbf{j}+ \end{array}
$$

ee ope o \rightarrow j $-P_j$ P_j

$$
j \quad V_j \to V_j
$$

and e operator represented y the \times matrix in \bullet in \bullet in \bullet

$$
A_{\mathbf{j}+} \quad B_{\mathbf{j}+} \qquad \mathbf{W}_{\mathbf{j}+} \oplus \mathbf{V}_{\mathbf{j}+} \rightarrow \mathbf{W}_{\mathbf{j}+} \oplus \mathbf{V}_{\mathbf{j}+} \qquad \qquad \mathbf{Q}
$$

 f ee \blacktriangleright co \blacktriangleright \blacktriangleright \blacktriangleright en en

$$
-\{\{A_j \mid B_j \cdot \underline{\mathbf{A}}\} \mid \mathbf{z}_j \quad \mathbf{n} \quad \mathbf{n}\}
$$

ee n $-P_n$ $P_{n_{\mathbf{a}}}$ fene of x es ane en \ldots n in and e operators are organized as cot the matrix cot the matrix cot the matrix cot the cot the cot Let e e foon \mathbb{r} of e on e

 $1.$ The operator Ajde x the interaction on the subspace only since the subspace θ is A and θ W_j n $\qquad \qquad$ a nee en of edec $\qquad \qquad$ n

 \mathcal{P} feope o $\mathcal{P}B_j$ _{in an}d described the interaction et een external description of $\mathcal{P}B_j$ in and description extending on $\mathcal{P}B_j$ in and description extending on $\mathcal{P}B_j$ in and description extending on j nd co $\bm{\varphi}$ of each meed, e.g. $\bm{\varphi}$ ce $\bm{\mathsf{V}}_{\mathbf{j}}$ contains all the subspace $\bm{\mathsf{V}}_{\mathbf{j}'}$ j' j $e^{\mathrm{i}\theta}$

 \mathcal{I} e operator jog is an "and" erfedt the operator jog \mathbf{I}

\n
$$
\begin{array}{cccccccc}\n \mathbf{F} & \text{e} & \text{e} & \text{e} & \text{e} & \text{e} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \text{e} & \text{e} & \text{e} & \text{e} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} \\
 & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z}
$$

and

 \hat{d}^1

 $\frac{\mathsf{A}}{\mathsf{S}}$ ¹

 \hat{d}^2

 $\frac{\mathsf{A}}{\mathsf{S}}$ 2

 $\frac{\lambda}{d}$ ³

 $\frac{\mathsf{A}}{\mathsf{S}}$ 3

 \sim \approx

Contract Contract Contract

 \mathcal{F} e opeo job job e per edyce in \mathcal{F} j
k;k′ \leftarrow y j;k j;k′ y d dy

en ϕ of coefficients k;k^o with $N -$ 1, repeated pp c on of the for \bullet \bullet \bullet \bullet \bullet produces k j;k' y d dy
 $N-$ epe ed pp c on of e

j

k+ i+ ;m+ l+

$$
\underset{k;m}{\underset{j,l}{\overset{1}{\rightleftharpoons}}}\frac{\mathbf{k}}{k} \underset{k+m}{\overset{j}{\underset{k+1}{\overset{1}{\underset{1}{}\rightleftharpoons}}}}\hspace{.2cm} k+m\underset{k+1}{\overset{j}{\underset{1}{\underset{1}{}\rightleftharpoons}}}}\hspace{.2cm} k^{j}
$$

III.2 The Standard Form

 \mathcal{F} e and d for so and y eperating \mathcal{F} $V_j =$ j′>j W^j

nd considering for each secale expansion of ${Bj'}$, ${j \choose j}$ $j' > j$ 0

The Standard Form
\nand
$$
d\theta = a_0
$$
 and $y \Leftrightarrow c_0$ and $y^2 = c_0$ and $y^3 = c_0$
\n
$$
V_1 := \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix}
$$
\n
$$
V_1 = W_1
$$
\n
$$
= \begin{pmatrix} \frac{1}{2} & W_1 \rightarrow W_1 \\ \frac{1}{2} & W_1 \rightarrow W_1 \end{pmatrix}
$$
\n
$$
= \begin{pmatrix} \frac{1}{2} & W_1 \rightarrow W_1 \\ \frac{1}{2} & W_1 \rightarrow W_1 \end{pmatrix}
$$
\n
$$
= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix}
$$
\n
$$
= \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix} \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix}
$$
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= \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix} \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix}
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= \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix} \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix}
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= \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix} \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix}
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$$
\n
$$
= \begin{pmatrix} -M & 0 \\ -M & 0 \end{pmatrix} \begin{pmatrix
$$

 $\mathcal{O}(\mathcal{O}(\log n))$

 $\mathcal{O}(\frac{1}{2} \log \frac{1}{2} \log \frac{1}{2}$

If there is the coarsest scale n then instead of (3.25), e ha e

$$
V_j = V_n \frac{J \hat{M}^n}{j^{\prime} j^+} W_j
$$

 $\lim_{n \to \infty}$ are equivalently are the same as in a same and same as in a same if nd n dd on for each $f \circ g$ ere eoperators ${B}^{n+}$ } and ${n+1}$ }

$$
B_j^{n+} \quad \mathbf{V}_n \to \mathbf{W}_j
$$

$$
\sum_{j=1}^{n+} \quad \mathbf{W}_j \to \mathbf{V}_n
$$

n ano on n^+ - ρ nd B_n^{n+} - B_n fen e of x ea ne nd V ned en on ϵ en ϵ and d form is representation of $-P$ P as

$$
-\{A_j \quad \{B_j^j'\}_{j'}^{j'} \quad \underset{\text{A}}{\mathbf{n}} \quad \{\underset{\text{A}}{\underset{\text{A}}{\mathbf{j}}}\}_{j'}^{j'} \quad \underset{\text{B}}{\mathbf{n}} \quad \{\underset{\text{A}}{\underset{\text{B}}{\mathbf{n}}}\}_{j'}^{j'} \quad \underset{\text{B}}{\mathbf{n}} \quad \underset{\text{C}}{\mathbf{n}} \quad \underset{\text{C}}{\mathbf{n}} \quad \underset{\text{C}}{\mathbf{n}} \quad \underset{\text{C}}{\mathbf{n}} \quad \underset{\text{C}}{\mathbf{n}} \quad \underset{\text{D}}{\mathbf{n}} \quad \
$$

 T e operators \bullet and \bullet organized as \bullet organized as blocks of the matrix \bullet \bullet \bullet \bullet /R36 (3aV)Tj /R360 Td (Figu65 ך

d **V**
3 C f

contributed on the CFigu65

contributed on the CFigu65

e co p e^{33} on of ope o³

 T e compression of operators order ords, the construction of the space representations in order orthonormal bases, \mathbf{c}_s and \mathbf{c}_s are speed of computational algorithms. \bigwedge e_s e cope_r and of legible peceed y end₂₀ e m $\text{ind} \text{ in } \mathbb{R}$ is equation in some basis may expect on some applications, the some application in some applications, the some applications, the some application in some applications, the some applications, the some ap compression of operators \bullet and \bullet representation in basis in order to effectively compene \clubsuit storte standard and θ and θ of operators in the electron \clubsuit \clubsuit y e e d \clubsuit compression \clubsuit e e \clubsuit for θ de c \clubsuit of θ and θ and θ of operators in the θ the matrices $\left| \begin{array}{ll} \cdot & j \end{array} \right|$, i (3.16) - (3.18) of the non-standard form satisfy the estimate

$$
\begin{array}{c|c|c} \mid & \mid & \mid & \mid \\ \mid & \mid & \mid & \mid \\ \mid & \mid & \mid & \mid \end{array}
$$

for all $|\cdot - \cdot| \geq M$.

Similar considerations apply in the case of pseudo-differential operators. Let e pseudo-differential operator with sym ol x, ξ defined y the formula f)(= e ix x, ξ ˆf ξ dξ = x, y f y dy (4.8)

ee \rightarrow ed \rightarrow on ene of

Proposition IV.2 If the wavelet basis has M vanishing moments, then for any pseudodi erential operator with symbol of and of satisfying the standard conditions

$$
|\begin{array}{ccc} & \times & \prec & \leq C \\ & \times & & \prec & \leq & C \\ & \times & & \prec & \leq & C \\ \end{array}; \qquad |\begin{array}{ccc} \downarrow & & + & & \circ \\ \downarrow & & + & & \circ \\ & \downarrow & & & \circ \\ & & & & \circ \\ \end{array}
$$

the matrices (i, j, j) (3.16) - (3.18) of the non-standard form satisfy the estimate

$$
\begin{array}{c|c|c|c} \mid & j & j & j \\ \hline i_{i} & -\frac{i}{i} & -\frac{i}{i} \end{array}
$$

for all integer λ , λ .

f e ppo e e opeo N y e opeo N ;B o ned fo N y \bullet n^o o ze o coe c en \bullet of ce i;l i_il nd i_il o **a**de of nd of d $B \geq M$ ond ed ion are easy open

$$
\|\mathbf{N} \cdot \mathbf{B} - \mathbf{N}\| \le \frac{C}{B^{\mathbf{M}}} \quad \text{of}^{\clubsuit} \quad N \tag{2}
$$

e e C is constant determed y the ernel In \mathfrak{O}_n and \mathfrak{e}_n constants, the error of \mathfrak{O}_n and \mathfrak{e}_n and \mathfrak{e}_n applications, the error of \mathfrak{O}_n and \mathfrak{e}_n and \mathfrak{e}_n and \mathfrak{e}_n and cc cy of c c on λ and the parameters of the algorithm (in our case, and the algorithm (ii) λ case, and ca e nd d B nd ode M e oeco \bullet en n \bullet c mae e de \bullet ed pecion of ccalculations is a celectric in B is a celectric on B is a celectric on B is a celectric on B

$$
|| \quad \mathbf{N} \cdot \mathbf{B} - \quad \mathbf{N} || \leq \frac{C}{B^{\mathbf{M}}} \quad \text{of}^{\clubsuit} \quad N \leq
$$

y = (1)(y (4.25) It is remarkable fact that y analysing the functions (4.24) and (4.25) (and, therefore, the operators L and L), it is possible to decide if Calder´on-Zygmund operator is ounded.

Theorem IV.1 (G. David, J.L. Journe) Suppose that the operator (3.1) satis es the conditions (4.5) , (4.6) , and (4.16) . Then a necessary and sucient condition for to be bounded on L is that in (4.24) and y in (4.25) belong to dyadic BMO , i.e. satisfy condition Z.

$$
\mathbf{a}_1^{\mathbf{p}} \overline{\mathbf{a}_1 \mathbf{b}_2} \mathbf{b}_3 \mathbf{c}_4 \mathbf{d}_5 \mathbf{c}_7
$$

where is a dyadic interval and

$$
\mathbf{z} \quad \mathbf{z} \quad
$$

 \mathbf{p} in the operator into the sum of the separately leads to the estimate of the estimations of the functions of \mathbf{q} and \mathbf{q} are \mathbf{q} $e \rightarrow y$ computed in the process of construction \cdots and f construction f and f and opode and e and e and e of the norm of the operator.

e d^rie en λ operators in elet δe^{λ}

V.1 The operator d=dx in wavelet bases

 \mathcal{F} e non-standard forms of several computed operators may explicitly explicitly recomputed explicitly may explicitly recomputed explicitly recomputed explicitly recomputed explicitly recomputed explicitly recomputed \therefore a econa c enon-standard form of experiment of the \mathbf{F}_t equator of the matrix elements in $\begin{bmatrix} 1 \end{bmatrix}$ of $\begin{bmatrix} 1 \end{bmatrix}$ of il $\mathbf{e} \cdot \mathbf{g} = \mathbf{f} \cdot \mathbf{g}$ and $r_{\mathbf{u}}$ of $\mathbf{g} = \mathbf{g}$ of $\mathbf{f} \cdot \mathbf{g}$ of $\mathbf{d} \cdot \mathbf{d}$. **V.1** The operator d=c
 \therefore c non and d for a c non and
 \therefore e ena c e non and
 $\frac{1}{4}$ of A_1 B_1 $\therefore A_2$ nd r_1 of ^j tsjd/dx el462 ele $\frac{3e^3}{e^4}$

es
 $\frac{0}{4}$ d' $\frac{7}{4}$ ero pedeport
 $\frac{1}{2}$ et d'add-d $\frac{1}{4}$
 $\frac{1}{2}$ d'add-d $\frac{1}{2}$

ee_n ee oco e on coecentof ee $-{\kappa}$ k L

$$
n = \frac{L \times n}{i}
$$

 \neq e \rightarrow e that the autocorrelation coefficients n with even indices are zero, \rightarrow

$$
\mathbf{k} = \mathbf{I} -
$$
\n
$$
\mathbf{A} \mathbf{y} = \mathbf{y} \mathbf{y} + \mathbf{y} \mathbf{y} \mathbf{y} + \mathbf{y} \mathbf{y} \mathbf{y} + \mathbf{y} \mathbf{y} \mathbf{y} \mathbf{y} + \mathbf{y} \
$$

$$
r^{\frac{1}{2}} \leq | \qquad - \qquad - \qquad \sum_{n} \quad n \, \cos n \leq
$$

|m ξ | = − L=k=1 ^k cos(2 − 1)ξ L=k=1 ^k cos ξ (5.22) where ⁿ are given in (5.19). Combining (5.21) and (5.22) to satisfy (2.26), e obtain

$$
\begin{array}{c}\n\mathbf{L}_{\mathbf{X}} \\
\mathbf{k} \\
\mathbf{k}\n\end{array}
$$

nd ence, and α \mathcal{F} ee en oen poof e coe cen p k for anish, namely

$$
\begin{array}{cccc}\n\mathbf{k} \mathbf{k}^{\mathbf{t}} & \mathbf{k} & - & \mathbf{m} & - & \text{for} & \mathbf{k} \\
\mathbf{k} & & \mathbf{k} & - & \mathbf{m} & - & \text{for} & \mathbf{k} \\
\mathbf{k} & & & & & \n\end{array}
$$

 \clubsuit nce

$$
-\begin{array}{c}\nm\\
\end{array}
$$

 α follows from the computation of the state α of α α of α

 $\bullet \quad n^*_{t} = \mathcal{P} \quad \text{e e} \quad e \quad 7 \quad \bullet$ $r_{\mathbf{i}}$ \mathbf{I} LX \mathbf{k} k \mathbf{x}_+ m k k k m r i+m

C $n^{\bullet} n^{\bullet}$ eode of \bullet on n and $\bullet n^{\bullet}$ efc \bullet \bullet \bullet \bullet e arrive at L

$$
r_1 = r_1
$$
 n r l n r l+n
$$
\mathbf{r} \in \mathbf{Z}
$$

ee n e[†] en n
node oo n

$$
\mathbf{r} \qquad \mathbf{r} \qquad \mathbf{r}
$$

l=+l −∞ l ^m [−] ^l ⁼ m lXm l=1 −1) ^l m l M 'l m l

e e

$$
M_{\mathbf{l}} = \begin{bmatrix} \mathbf{L} & \mathbf{L} \\ \mathbf{L} & \mathbf{L} \end{bmatrix} \mathbf{d} \qquad \mathbf{L} = \mathbf{M}
$$

ee oen ϕ of efficon $\overline{5}$ on to ϕ as pyon n^o oe m to and m^r Leibniz e and m^r and m with m e obtain $f M > \text{en}$

$$
|\mathbf{1} \cdot \mathbf{1} \cdot \mathbf{1}| \cdot \mathbf{1} \leq C \qquad |\mathbf{1} \cdot \mathbf{1}|
$$

ee ndence end^s n \neq p eyeon ergent. This assertion footo Lemma 3.2 of \log and \log \log \log

$$
|\mathbf{1}_{\mathbf{1}_{\mathbf{1}}}\mathbf{1}_{\mathbf{2}_{\mathbf{2}}}\leq C\qquad |\mathbf{1}_{\mathbf{1}_{\mathbf{1}}}\mathbf{1}_{\mathbf{1}_{\mathbf{1}}}\mathbf{1}_{\mathbf{2}_{\mathbf{2}}}\mathbf{1}_{\mathbf{2}_{\mathbf{2}}}\mathbf{1}_{\mathbf{2}_{\mathbf{2}}}
$$

e e

$$
B - \bullet \mathbf{p} | \quad \mathbf{e}^i |
$$

 $D e o e e cond on e e o f B = M -$

 \mathcal{F} e.e. spence of ϕ solution of existence of equations (5.14) and (5.14) and (5.14) and (5.14) ϕ for ee ence of enger in the existence of the integral in \mathbb{R} in \mathbb{R} of the scaling function has compact \clubsuit pport ee eonly find that $\frac{\pi}{2}$ for $\frac{d}{dx}$ and $\frac{d}{dx}$ are there.

 \sim

∞@∞∈{∈{∞}∞¢∞∈{∞∈∞∞|∞|∈ 5894 0ed e e

$$
r \leqslant \frac{X}{1 - r_1 e^{i\theta}}
$$

$$
r_{\text{even}} \leqslant \frac{1}{1 - r_1 e^{i\theta}}
$$

and

$$
r_{\text{odd}} \leftarrow \begin{array}{c} \mathbf{X} \\ -r_{\mathsf{I}+} e^{\mathsf{i} \left(\mathsf{I}+ \right)} \end{array}
$$

No c n^*

$$
T_{\text{even}} \leqslant \quad -T \leqslant \quad T \leqslant
$$

and

$$
r_{\text{odd}} \prec \qquad -r \prec \qquad -r \prec
$$

nd \mathbf{m}^s e o n f o

$$
r \leqslant -r \leqslant r \
$$

 $\ln y$ and $\ln z$ e e

$$
r \prec - \not \rightarrow r \prec + r \prec \not \rightarrow r \prec + r \prec
$$

e n^o ϵ = n e eo nr $-r$ nd and

In q energies of exponding ind to \circ and the solution of \bullet in q energies of the representation of d d density of \mathbf{g} and \mathbf{g} and \mathbf{g} and \mathbf{g} and \mathbf{g} e consider the solution representation representation representation representation representation representation represen ope o j de ned y execce centron expectification of \mathbf{v}_j and p \clubsuit oo f nc on f nce $r_1^j = {^j}r_1$ g e e

$$
\mathbf{y}f = \begin{bmatrix} \mathbf{x} & \mathbf{y} & \mathbf{x} \\ \mathbf{x} & \mathbf{z} & \mathbf{y} \\ \mathbf{x} & \mathbf{z} & \mathbf{y} \end{bmatrix} \mathbf{x}_{r_1 f_{\mathbf{y}; \mathbf{k}} + \mathbf{y}; \mathbf{k}}
$$

where

$$
f_{\mathbf{j},\mathbf{k}+1} = \begin{array}{ccc} \mathbf{j} & \mathbf{k} & \mathbf{j} & \mathbf{k} \\ \mathbf{k} & \mathbf{k} & \mathbf{k} \\ \mathbf{k} & \mathbf{k} & \mathbf{k} \end{array} \qquad \qquad \mathbf{j} \quad \mathbf{k} \qquad \qquad \mathbf{k} \q
$$

 $\int_{\mathcal{C}}$ as $\int_{\mathcal{C}}$ as $\int_{\mathcal{C}}$

$$
f_{\mathbf{j};\mathbf{k}+\mathbf{1}} =
$$

j

d \mathcal{F} \mathcal{F} 7.9

 $\begin{array}{ccccccccccccc} \mathrm{e} & \mathrm{e} & - & - & \mathrm{j} & \mathrm{nd} & - & \leq & \mathrm{j} & & \end{array}$ and $\begin{array}{ccccccccccccc} \mathrm{e} & \mathrm{e} & - & - & \mathrm{j} & \mathrm{nd} & - & \leq & \mathrm{j} & & \end{array}$ nd e o n

$$
\mathbf{j}f = -\frac{\mathbf{x} \cdot \mathbf{z}}{\kappa z} + f' \quad \mathbf{j} \cdot \mathbf{k} \quad d \quad \mathbf{j} \cdot \mathbf{k}
$$
\n
$$
\mathbf{j} \times \mathbf{x} \times \mathbf{x} \times r\mathbf{l} \cdot \mathbf{z} + f'' \quad \mathbf{j} \cdot \mathbf{k} \quad d \quad \mathbf{j} \cdot \mathbf{k} \qquad \mathbf{q} \tag{7}
$$

 \clubsuit ce \clubsuit \rightarrow $-\infty$ operators j and d/d coincide on \clubsuit oo fine on \clubsuit and **a**nd d \mathbf{r} en a secondo e that oppose that $-d\,d$ and ence, e appoint of \mathbf{e} and \bullet nge fee on follows no fol

Remark 2 \oint e no $e_{\frac{1}{2}}$ e pezons (5.9) and (5.10) for l and l l l = - l may e aped y changing the order of a summation in the order of summation in \mathbf{S} and integration in \mathbf{S} e co e on coe c en λ $\begin{bmatrix} 1 & n \\ i & i+n \end{bmatrix}$ i i+n and $\begin{bmatrix} 1 & n \\ i & i+n \end{bmatrix}$ i i+n λ e epezon for la pecially simple, l $\frac{1}{4}$ r l $\frac{1}{4}$ r l $-$ rl

Examples. oe e peare a propose \bullet Daubechies' electric in \bullet e acope ecoe compared the M ee M and \mathbf{m} is the num error of \mathbf{n} and o en \clubsuit nd $L - M$ \clubsuit nd^r e on e of j.

$$
\mathbf{P}^{\dagger} \quad \mathbf{P}^{\dagger} \quad = \quad -\frac{M - \mathbf{M}}{M - \mathbf{M}} \quad \mathbf{M} \quad \mathbf{P}^{\dagger} \mathbf{M} \quad \mathbf{P}^{\dagger} \mathbf{M}
$$

end y copusⁿ an^M \lll

$$
\mathbf{y}^{\sharp} \quad \mathbf{y}^{\sharp} \quad \mathbf{y}^{\sharp} \quad \mathbf{y}^{\sharp} \quad -\quad \mathbf{
$$

e e

$$
C_{\mathbf{M}} = \frac{M - \frac{\mathbf{#}}{M}}{M - \frac{\mathbf{A}^{\mathbf{M}}}{\mathbf{A}^{\mathbf{M}}}}
$$

 $T \rightarrow y \text{ co } p \text{ m}$ e had e e

$$
m \quad - \frac{-m}{M} \frac{C_M}{r^4} \qquad M \quad r^4 \quad - \quad r^4 \quad - \qquad e \quad e \quad r^4 \quad - \qquad M
$$

 \bigwedge^{\bullet} note that y viewed \bigwedge^{\bullet} solutions of linear system with rational coe cen λ _n n e on y construction e coeffen r_1 e on n e \blacktriangleright fecoecentre e \blacktriangleright eforthe same for all e of $\mathfrak n$ and $\mathfrak m$ or entropy momentum coefficient basic from M where $\mathfrak s$ are seen basic from M

j e
$$
\int d\uparrow
$$
 \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow

$$
\mathbf{j} \qquad \text{ene e} \qquad \qquad \mathbf{i}_0 \qquad \mathbf{y}_{\text{no}} \qquad \mathbf{y}_{\text{r}}
$$

on \mathbb{S} eq on sof opositon the results for Daubechies' electric equation is electrical elements. $M = 9$ 1 $M =$

$$
\mathcal{L}^{\frac{1}{2}}\left(\mathcal{L}^{\frac{1}{2}}\right) = \mathcal{L}^{\frac{1}{2}}\left(\mathcal{L}^{\
$$

and

$$
r \sim - -
$$

 $T \text{ e coe } \text{ e en } \blacktriangleright -$ 12 of \blacktriangleright example can e found in my ooks on numerical analysis as choice of coefficients and \mathbf{F} control \mathbf{F} and \mathbf{F} and \mathbf{F}

5 $M -$

Coecentro M = and M = can e corpared with the corresponding M op foefoon \mathbb{I} ee \mathbb{I} o

Iterative algorithm for computing the coecients r_1 .

As y of some A solving A and B and B and B and B and B and B a^r \uparrow a^r a^r r_1 are ay one fy and α is and (5.17) and (5.17) and (5.17) and (5.17) are satisfied due to the choice of n z on $f \circ f$ on f' folloce to be \bullet as $M = 7$ coped sn^{\bullet} a r_0 d p_0 y e coe cen $\text{a}_{\{r_1\}}^{\text{L}}$ ve note, that r is $-r_1$ nd $r =$

V.2 The operators d^n =dxⁿ in the wavelet bases

o eope odd enon \clubsuit nd dfoof eope odⁿ dⁿ \clubsuit coperator de operator de dee ned y \Rightarrow epe \Rightarrow n on on e \Rightarrow \Rightarrow ce V e y e coe c en \Rightarrow

$$
r_1^{(n)} = \begin{cases} \frac{d^n}{d^n} & d \\ t \in \mathbb{Z} \end{cases}
$$

o en ey

$$
r_1^{(n)} = \begin{vmatrix} z_+ \\ -\epsilon^n \end{vmatrix}, \quad z_+^{(n)} = e^{i\theta} d\epsilon
$$

from $e \neq e$ and $e^{i\theta} e$

Proposition V.2 1. If the integrals in (5.52) or (5.53) exist, then the coecients r_1 ⁿ⁾ \mathbf{p}^{H} , $\mathbf{p} \in \mathsf{Z}$ satisfy the following system of linear algebraic equations

$$
r_1^{(n)} = \frac{1}{2} \sum_{k=1}^{n} \frac{1}{k} \sum_{k=1}^{n} \frac{1}{k+1} \sum_{k=
$$

and

$$
\sum_{i=1}^{n} r_i^{(n)} = -n
$$

where k are given in (5.19).

2. Let $M \geq n$, where M is the number of vanishing moments in (2.16). If the integrals in (5.52) or (5.53) exist, then the equations (5.54) and (5.55) have a unique solution with a $\,$ nite number of non-zero coe $\,$ cients $r_{1}^{\,}$ $\,$ $\,$ $n^{(n)}$, namely, $r_1^{(n)}$ $\mathbf{I}^{\mathbf{m}}$ for $-L \leq L \leq L - 1$. Also, for even n

$$
r_1^{(n)} - r_1^{(n)}
$$
\n
$$
\times \quad r_1^{(n)} - n - n - 7
$$
\n
$$
\times \quad r_1^{(n)} -
$$

l

and

and for odd n

$$
\begin{array}{ccc}\n & r_1^{(n)} & -r_1^{(n)} \\
\mathbf{x} & \mathbf{n} & r_1^{(n)} & -r_1^{(n)} \\
1 & \mathbf{v} & & \mathbf{v} & \mathbf{v} & \mathbf{v}\n\end{array}
$$

 \oint e noe on \int e ee \blacktriangleright e $L =$ e ee \blacktriangleright o n \blacktriangleright n^{\blacktriangleright} o en $\clubsuit M =$ do not have the exponent xe_{\clubsuit} , but the representation of edde ee \clubsuit ony fen eof n \clubsuit n l^* oen $\clubsuit M = 2$

 \mathcal{F} e equations for computing the coefficients represents respectively. The coefficients respectively on \mathbf{F} end \mathbf{F} poe Le \rightarrow dee eeq on coe \rightarrow pondin $^{\prime\prime}$ oe for d ⁿ die y for \leftarrow e e \rightarrow

$$
r_1^{(n)} = \begin{array}{c} Z & X \\ k & Z \end{array} \qquad | \quad n \leqslant \qquad \begin{array}{c} n_e \text{ if } d \leqslant \end{array}
$$

 $\mathcal F$ e efo ${\rm e}$

$$
r \prec \frac{\mathbf{x}}{\mathbf{k} \mathbf{z}} \vert, \ \prec \qquad \vert \ \ ^n \prec \qquad \ ^n
$$

e e

$$
r \preccurlyeq -\frac{\mathbf{x}}{1} r_1^{n} e^{il}
$$

 \bullet m^* e e on

 \cdot \leq $\overline{\gamma}$ \leq \cdot \leq no e^r ind_ride of ind_ride \ln ^o oe een nd odd indice_s in \bullet p e y e e

$$
r \prec - \sqrt{r} \not\sim \sqrt{r} \prec \sqrt{r} \prec \sqrt{r} \prec \sqrt{r}
$$

Let Riemannian exconsider the operator M on -periodic functions d f n

$$
Mf \prec \overline{\mathbf{r}}^{\prime} \prec f \prec \mathbf{r}^{\prime} \prec \mathbf{r}^{\prime}
$$

 f d

in ee \Rightarrow en ec e dence \Rightarrow numerical episodinees interest. example of ed n lexof copular n e ee \rightarrow \mathbb{F}_2 ¹ ece \mathbb{F}_2

e con ol $\sqrt[4]{\text{on ope}}$ o³ n ele³e³

 $n \rightarrow \text{ec on}$ e consider the computation of the non-standard form of contact of ope o \clubsuit o con o on operators the quadrature formulation \bullet for experimulation of \bullet representing the ernel on **V** e of e \rightarrow pe \rightarrow for due

nd e den $y \cdot \cdot \cdot = \cdot \cdot \cdot p^q$ $\cdot \cdot$ \neq \cdot follows from \bullet \circ d₂.

 $\frac{d\mathbf{F}}{d\mathbf{r}}$ e oen and e function and \mathbf{F} equation \mathbf{F} equation (6.4) pong denote formula for computing the representation of con one of α e nest scale for all compactly supported electric \bullet of \bullet \bullet $e \rightarrow e$ as f to the special choice of the special special e eqns. ee e \neq fed o en \neq of efnc on $n \neq$ e efe $e \text{ de } \blacktriangleright$ $\frac{4\pi^2}{\pi^2}$ of the case of π is π

 \blacksquare e e n od ce differen ppo c consists in solving the system of near algebraic equations \bullet as equations (6.2) subject to assumpt to assumptions. The intervals of \bullet is especially subject to assumptions. The intervals of \bullet is equations. The intervals of \bullet is equations. The i cyppe fegy out expedience of \mathbf{e} of \mathbf{e} of \mathbf{e} of \mathbf{e} of \mathbf{e} in the integree since in case the operator is completely defined y its representation on $\mathbf V$

Le \triangle con \triangleleft de o epe \triangleleft of \triangleleft copeo \triangleleft ope of fractional \bullet or and \bullet and \bullet and \bullet and \bullet

VI.1 The Hilbert Transform

 \bigwedge^e ppyo e od o e cop on of e non \bigtriangleup nd d form of the non-standard trans $f_{\rm O}$

$$
-\mathcal{H}f \quad y \quad --\quad p \qquad \frac{f}{-}d
$$

e e p
 $\;$ deno $e\spadesuit\;$ p
 nc p $\;$ $\;$ e $\;$

 \mathcal{F} e eperne on of H on **V** is defined y e coefficients

$$
r_1 = \qquad \qquad -\frac{1}{t} \qquad \mathcal{H} \qquad d \qquad t \in \mathbf{Z}
$$

c n n cope eyde ne oe coe centos of enon \clubsuit nd d 4635)Td (0)Tj / / / /)Tj /R33km,7 Td Td (s)Tj 2355279 EI Q

 \mathcal{F} eg \mathcal{F} e coe $_{\mathbf{a}}$ c en $\mathbf{a}r_1$ of e n for b D ec e a ee λ and λ and λ

$$
\mathbf{a} \quad \mathbf{b} \quad \mathbf{c} \quad
$$

$$
r_{1} = r_{1} - \frac{\mathbf{k}}{k} \qquad r_{1} = k + \frac{r_{1+1}}{k} \qquad \qquad \mathbf{q}
$$

ee ecoecentrs k are given in \mathbb{Z}^n and \mathbb{Z}^n equal to be and (6.6) and (6.6) and (6.7) e obtains and (6.7) e obtains a set of \mathbb{Z}^n and (6.7) e obtains a set of \mathbb{Z}^n and (6.7) e obtains a set of $\mathbb{$ e \rightarrow poctor_l for te_{\rightarrow}

$$
r_1 \longrightarrow 0 \frac{1}{\pi}
$$

By e n^* n e \rightarrow of \leftarrow of \leftarrow $\$

e o $\ln r_1 = -r_1$ and \bullet $r_2 = \bullet$ one that the coefficient r c note th foeq on_{\bullet} e nd

on \mathbb{R} on \mathbb{R} with the asymptotic condition (6.15), e compute the coefficients of \mathbb{R} r_1 $r \neq$ hypered accuracy

Example.

 $\oint e$ computer the coefficients right transform for Γ the Γ the Γ of Γ and Γ of Γ and Γ of Γ and Γ of Γ of Γ and Γ of Γ with six and \mathbb{R} and \mathbb{R} and \mathbb{R} are obtained by \mathbb{R} are obtained by \mathbb{R} are obtained and a pocal (enoe $r_1 = -r_1$ and $r = 0$).

VI.2 The fractional derivatives

 \bigwedge^e **e** foon^{\bigwedge^e} denon of f c on dee \bigtriangleup

$$
\mathbf{x} \mathbf{f} \qquad \mathbf{I} = \begin{bmatrix} \mathbf{Z} & & & & \\ & & & & \\ \hline & & & & \\ & & & & \\ \hline & & & & \\ & & & & \end{bmatrix} \qquad \mathbf{f} \quad y \, dy \qquad \qquad \mathbf{f} \qquad \mathbf{f
$$

e e consider \neq If \qquad en \qquad defines fractional anti-derivatives. \mathcal{F} e epe \mathbf{p} on of \mathbf{x} on \mathbf{V} and \mathbf{v} are coefficients.

$$
r_1 = \begin{cases} 2 & \text{if } x & \text{if } d & \text{if } x \in \mathbb{Z} \\ 1 & \text{if } x & \text{if } d & \text{if } x \in \mathbb{Z} \end{cases}
$$

poded ϕ ne e ϕ

 \mathcal{F} e non-standard form x = {Aj Bj \mathcal{F} j z standard e d A j = jA Bj = $\mathbf{J}B$ and $\mathbf{J} = \mathbf{J}$, ee ee $\oint \mathbf{\hat{n}}$ if it and in of A B and \mathbf{J} eo ned f^roe e coefficients r_1

$$
\mathbf{K} \mathbf{K}
$$
\n
$$
\mathbf{K} \mathbf{K}
$$

and

$$
\mathbf{K} \mathbf{K}
$$
\n
$$
\mathbf{K} \mathbf{K}
$$
\n
$$
\mathbf{K} \mathbf{K}
$$
\n
$$
\mathbf{K} \mathbf{K}
$$

egy oe fy ecoe central that the following system of ne \mathcal{P}_e c eq on \mathcal{P}

$$
r_{1} = \begin{array}{ccccc} & & & & 3 \\ 4r_{1} & - & k & r_{1} & k+ & r_{1+k} & 5 \\ k & & & & & \end{array}
$$

 \mathbb{R}^3 e nd $\begin{array}{ccc} 7 & \text{e o} & \text{n} \end{array}$ e \rightarrow poctor_l for te_{\rightarrow}

$$
\begin{array}{ccc}\nr_1 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
r_1 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
r_1 & r_2 & r_1 & r_2 & r_1 \\
r_2 & r_2 & r_2 & r_2 & r_2\n\end{array}
$$

Example.

$M \psi$ ¹ ψ on of ope o³ n ele³ e^3

VII.1 Multiplication of matrices in the standard form

 \mathcal{F} e pc on of ceptoof C de on-Zy \mathcal{F} and nd ppe do-different operators in e.g. nd d form equipe at a post $O N o^{\bullet} N$ goperations. In addition, it is possible o control the defect of the $\mathbf{d} \bullet \mathbf{y}$ is $\mathbf{d} \bullet \mathbf{y}$ and $\mathbf{d} \bullet \mathbf{y}$ and $\mathbf{d} \bullet \mathbf{z}$ in the entries in the product of ent eo esod of

nd e efo e

$$
7\,\mathrm{}
$$

 $|| \cdot - \cdot || \leq$ $\mathcal{F}_{\mathcal{R}}$ \mathcal{P} and \mathcal{A} de of (\mathcal{A} and \mathcal{A} \mathcal{C} op one single c n digit.

VII.2 Multiplication of matrices in the non-standard form

 $\oint e$ no one n Γg for the ultiplication of the operators in the nonand d for \mathcal{S} ane \mathcal{S} or \mathcal{S} remarkable in the scale in the scales in the sc t_{p} or p_c on $\text{L}\text{e}_\text{c}$ and e o operators

$$
\mathbf{L} \quad \mathbf{R} \rightarrow \mathbf{L} \quad \mathbf{R} \tag{7.7}
$$

en e non \clubsuit nd d fo \clubsuit of · nd $\{A_j \ B_j : j \}$ j z nd $\{A_j \ B_j : j \}$ j z e com peenon-the discrete the $\{A_j \ B_j \ A_j\}$ j z of $-\frac{z}{z}$ $\oint e \quad ec \qquad \qquad e \quad ope \quad o \not\Rightarrow of$

Finally e rewrite (7.10) as sum of o terms, ˆ = (7.11)

e e

$$
\int_{-}^{\mathbf{j}\mathbf{x}^{\mathsf{n}}\mathbf{h}} A_{\mathbf{j}} A_{\mathbf{j}} B_{\mathbf{j} \cdot \mathbf{n}} B_{\mathbf{j} \cdot \mathbf{j}} A_{\mathbf{j}} B_{\mathbf{j}} \cdot A_{\mathbf{j}} B_{\mathbf{j}} \cdot A_{\mathbf{j}} A_{\mathbf{j}} \cdot A_{\mathbf{j}} A_{\mathbf{j}} \cdot A_{\mathbf{j
$$

 $\mathop{\hbox {nd}}$

$$
P_j: \mathcal{A}^B P_j
$$

$$
\begin{array}{ccccccc}\n\mathbf{\hat{f}} & \text{e} & \text{e} & \mathbf{a} & \mathbf{a} & \mathbf{b} \\
\mathbf{f} & \text{e} & \mathbf{a} & \mathbf{b} & \mathbf{c} \\
 & \mathbf{a} & \mathbf{a} & \mathbf{b} & \mathbf{c} \\
 & \mathbf{a} & \mathbf{a} & \mathbf{b} & \mathbf{b} \\
 & \mathbf{b} & \mathbf{b} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{b} & \mathbf{b} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{b} & \mathbf{b} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{b} & \mathbf{b} & \mathbf{c} & \mathbf{d} & \mathbf{b} \\
 & \mathbf{b} & \mathbf{b} & \mathbf{c} & \mathbf{d} & \mathbf{b} \\
 & \mathbf{c} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{c} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{c} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{c} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{c} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{c} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{c} & \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{c} & \mathbf{d} \\
 & \mathbf{c} & \mathbf{b} & \
$$

$$
\mathbf{e} e_{\mathcal{I}} = n \qquad \qquad \mathbf{A}^T \mathbf{f}_d \qquad \qquad \mathbf{A}^T \mathbf{f}_d \mathbf{g} \qquad \qquad \mathbf{A}^T \mathbf{f}_d \qquad \qquad \mathbf{A}^T \mathbf{f}_d \qquad \qquad \mathbf{A}^T \mathbf{g} \qquad \qquad \mathbf{A}^T
$$

of operations is halved each energy is the space of the space \mathbf{a} is \mathbf{a} and \mathbf{b} and \mathbf{c} and \mathbf ope on \clubsuit at \clubsuit ope on o N \mathcal{F} e eم n \mathcal{T} operators A j B j j j \mathcal{F} tot49 -410.315ac5 0 Td (36su8712.7097 0552 /R36 ^j

$$
\begin{array}{ccccccc}\n\mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{c} & \mathbf{d} & \math
$$

VIII.1 An iterative algorithm for computing the generalized inverse

n o de o

pocedendee on heep \mathbf{a} for \mathbf{f} fecop on eeperformed on \mathbf{p} and e used one foll N AC for computing the singular alue decomposition. \circ ests experience also established by \bullet \triangleleft e foon^s full

$$
A_{ij} = \sum_{\lambda}^{8} \frac{1}{1} \lambda \neq 0
$$

eet, $j = N$ is executed as set to 10^4 e entries N_k equal to 10^4 4 ee $\rightarrow \bullet$ e cyeoed feec e on

Le \bigtriangledown describe several iterations in the several function \bigtriangledown cc_a opeo in e peenede cen year for parto differential ope $o \blacktriangle$ N e c e \rightrightarrows and e e performe of e \blacktriangleright \uparrow o a e reported \bullet y ey

VIII.2 An iterative algorithm for computing the projection operator on the null space.

Le a consider the following iteration

$$
X_{\mathbf{k}+} = X_{\mathbf{k}} - X_{\mathbf{k}}
$$

$$
X = A A
$$

 T en $-X_k$ con erges or Pnull T see n e shown either directly or y commit n n are $\mathbf{e} = \mathbf{e}$ and $\mathbf{e} = -\mathbf{A} + \mathbf{A} + \mathbf{A}$ with the interval the interval of $\mathbf{e} = \mathbf{e}$ and $\mathbf{e} = \mathbf{e}$ is the interval of $\mathbf{e} = \mathbf{e}$ is the interval of $\mathbf{e} = \mathbf{e}$ is the interval of \math cope et generalized ne A and A the fact up control control experience algorithm makes the fact up of \overline{a} and \overline{b} and \overline{c} and \overline{d} and \overline{c} and \overline{d} and \overline{c} and \overline{d} and \overline{c} and e on ϵ fast for declass for ϵ as the same complexity a δ for the general intervalse in the importance is over, the importance is over, the importance is a contract of δ . g does not eq the in example of the interpretation only of the potential only of the post e ope o

VIII.3 An iterative algorithm for computing a square root of an operator.

Let \clubsuit de \clubsuit en egnocons constructed $A = \text{nd } A = \text{ne } A \clubsuit$ for since A \clubsuit fdon ndnon-ne \blacksquare edeneopeo \lozenge e \cdot on \clubsuit de efoo \blacksquare eon

 $Y_{1+} = Y_1 - Y_1 X_1 Y_1$ X_{1+} X_{1} $Y_{1}A$ $Y \quad - \quad - \quad A$ $X = -\mathbf{A}A$ ee acoamp eftered eof A ae \mathbf{z}_k n \sqrt{k} \mathcal{F}_g and \mathcal{F}_g are X_l con erges to $A = \text{nd } Y_l$ or $A = \text{by}$ writing $A = \text{by}$ writing $A = \text{by}$ where \mathcal{F}_l

D is diffond v in yields of type X_1 and Y_1 can even $\lambda X_1 = V P_1 V$ and $Y_1 = V$ iv ee P_1 and led \mathcal{P} and and

^l+1 = ^l − ^lP^l ^l Pmakes the iteration (8.3)-(8.4) fastf,7011ψTf 6.8507ψ0ψTd (easy)A97ψTf 8.75117ψ0ψTd ())Tj /R33ψ11.9552ψTf 4.55491ψ0ψTd (,)Tj

VIII.4 Fast algorithms for computing the exponential, sine and cosine of a matrix

 \mathcal{F} e exponential of corresponding to a original or and cosine functions are and cosine functions are and cosine functions are and cosine functions are and cosmology expected and cosmology experiments are and cosmolo a on f to f and f or e considered in any calculated in \mathbf{a} of operators. As in the case of \mathcal{C} ene zed n e \mathcal{F}

X Cop^{λ}n $F(u)$ in e ele⁵e⁵

In \neq econede \neq e fad pe loop of ee angles is an equal of \bullet and \bullet is an equal of \bullet is in ney different efficion and \neq eperied in eelet basis. An inportant and in electric basis of \blacksquare e pe \blacktriangleright = \bigcirc nyce \blacktriangleright \blacksquare Tene ze eoe \blacktriangleright of MBony [10], [10], \Box , one pop^ron of \Box et \Box on \Box on \Box of non-neequations on \Box \mathbf{n}_{\bullet} , ecppoc oee anoe \bullet eepec de n \mathbf{r} eof ppc on a of a \mathcal{L}_{O}

IX.1 The algorithm for evaluating u^2

 \bigwedge^{\bullet} \bigwedge^{\bullet} and \bigwedge^{\bullet} on a algorithm to \bigwedge^{\bullet} compute \bigwedge^{\bullet} Let \bigwedge^{\bullet} project ∈ **L** R on a ap cea V_j ; ∈ Z ap

$$
j \cdot P_j \qquad j \in V_j
$$

no de ordecope e x es e e series ϵ e ϵ

$$
-\frac{\mathbf{i} \times^{n} \mathbf{h}}{\mathbf{j}} P_{j} - P_{j} \qquad - P_{j} \qquad - \frac{\mathbf{i} \times^{n} \times^{n}}{\mathbf{j}} P_{j} \qquad P_{j} - P_{j}
$$
\n
$$
\lim_{h \to 0} \int_{0}^{h} P_{j} P_{j} - P_{j} \qquad \text{for} \quad n
$$

j=1

 \overline{O}

$$
-\sum_{j}^{j} P_j \qquad j \qquad \sum_{j}^{j} \qquad \qquad \sum_{j}^{n} P_j \qquad \qquad \sum_{j}^{n}
$$

n g ee ano ne con e een differen a easy $\operatorname{nd} j' j \neq j'$ For e_n n e c pposes e need formulas (9.3) or entitle numerical purposes (9.3) with finite numerical purposes in entropy i of \mathcal{F} ex of \mathcal{F} ace

Before proceed in function of ϵ and ϵ in the Haar basis of $\oint e$ e e foonlepe e on

$$
\begin{array}{ccc}\n\mathbf{j} & \mathbf{j} & \mathbf{j} \\
\mathbf{k} & \mathbf{k} & \mathbf{j} \\
\mathbf{j} & \mathbf{j} & \mathbf{j} \\
\mathbf{k} & \mathbf{j} & \mathbf{k} \\
\mathbf{j} & \mathbf{j} & \mathbf{j} \\
\mathbf{k} & \mathbf{k} & \mathbf{k}\n\end{array}
$$

A oepod c_{\bullet} on e \bullet e \bullet ereo. pndn epc y no \blacksquare

j k

$$
\begin{array}{ccc}\n&\mathbf{X}^n \mathbf{X} & & \mathbf{X} & & \mathbf{n} & \mathbf{n} \\
\mathbf{X}^n \mathbf{X} & & \mathbf{X} & & \mathbf{n} & \mathbf{n} \\
\mathbf{j} & \mathbf{k} & \mathbf{Z} & & \mathbf{k} & \mathbf{Z}\n\end{array}
$$

and \bullet \bullet \bullet \bullet of \bullet \bullet

= jXn j=1 j= X k Z d j k j k j k jXn j=1 j= X k Z d j k j k n= X k Z n k n k

On deno \mathbf{n}^*

$$
d_k^j = \begin{array}{ccc} \mathbf{j} = & + & d_k^j & \mathbf{j} \\ \mathbf{j} & - & \mathbf{j} = & d_k^j \\ \mathbf{k} & - & \mathbf{n} = & \mathbf{n} \\ \mathbf{k} & \mathbf{k} & - & \mathbf{k} \end{array}
$$

e e \bullet

$$
\begin{array}{ccccccccc}\n&\mathbf{X}^n & \mathbf{X} & & & \mathbf{X}^n & \mathbf{X} & & & \mathbf{X} & & \
$$

 \oint e note that if the coecient d_k^j $\frac{1}{\mathsf{k}}$ is zero then there is no need to keep the corresponding average in oe ods e need oeep ereson ynear the singular test e e e e e e coe c en $\partial d_{\mathbf{k}}^{\mathbf{j}}$ \mathbf{j} op od c \mathbf{k} \mathbf{k} \mathbf{k} e^t n c n for \mathbf{r} en cc cy

of coe cen \bullet c need o e \bullet o ed y e ed ced function \bullet which \bullet for \bullet e pe

$$
M_{\mathbf{W}\mathbf{W}\mathbf{W}}^{\mathbf{j}\cdot\mathbf{j}'} = \int_{\mathbf{L}}^{\mathbf{L}} \mathbf{I}^{\mathbf{S}} = \begin{bmatrix} \mathbf{Z} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} \end{bmatrix}^{\mathbf{S}'} \qquad \mathbf{I}^{\mathbf{S}'} = \int_{\mathbf{K}}^{\mathbf{S}'} \mathbf{K} \mathbf{K}^{\mathbf{S}'} \qquad \text{and} \qquad \mathbf{I}^{\mathbf{S}'} = \int_{\mathbf{L}}^{\mathbf{S}'} \mathbf{K} \mathbf{K}^{\mathbf{
$$

 \clubsuit

$$
M_{\mathbf{W}\mathbf{W}\mathbf{W}}^{\mathbf{j},\mathbf{j}'}\mathbf{y} = \mathbf{J}' = M_{\mathbf{W}\mathbf{W}\mathbf{W}}^{\mathbf{j},\mathbf{j}'} = \mathbf{y} \qquad \qquad \mathbf{y} \qquad \mathbf{y}' = \mathbf{y}
$$

 \blacksquare o e e e o \clubsuit \blacksquare n e d c on n e n e of coe cen \clubsuit a conseq ence of e f c e coe c en λ n dec y λ e d λ nce r \rightarrow j' e een e \blacktriangleright e \blacktriangleright

f en e of \mathbf{a}^{\dagger} n cn coe cen $\mathbf{a}d_{\mathbf{k}}^{\dagger}$ ap opo on o en e of \mathbf{a} ea k of $N \not\rightarrow$ e en e of ope on eq ed o e e_s e pp n^e \clubsuit p c e and the store of \bullet is necessary to some only to see the state of \bullet is necessary to see th e \mathbf{z} n c n coe c en $\mathbf{z} d_k$ $\frac{1}{k}$ o pod ce non-ze o con on $\frac{1}{k}$ e efoc \Rightarrow cen o so e only $\circ \Rightarrow$ $\frac{1}{k}$ for choose ever due to d_k k λ c $| - | \leq \t nd \t e p o d c \t {d \t k'}$ \mathbf{k} above the threshold of accuracy is entropy to the threshold of \mathbf{k} need opoe e lepony nenel oood of \mathbf{m} era \mathcal{F} en eof opeon for epindin \mathcal{F} of expandent in order ee \leftrightarrow propo on o en e of \Box ncnene and ee equipment completely similar to the Haar basis of the Haar basis of the Haar basis of the Haar basis of the Haar basis o Remark. The loop algorithm for evaluation π in the elet basis allows us to $\lambda \geqslant 0$ e epodc of o functions, since $-\frac{1}{4}$. $-\frac{1}{4}$. $\begin{array}{r} \n\begin{array}{r}\n\text{e} & \text{if } x \in \mathcal{A} \\
\text{c} & \text{co} & \text{ne} \\
\text{d} & \text{d} & \text{ce} \\
\text{e} & \text{f} & \text{e} \\
\text{f} & \text{f} & \text{f} \\
\text{f} & \text{f} & \text{g}\n\end{array}\n\end{array}$
 $\begin{array}{r} \n\text{no} & \text{e} \\
\text{c} & \text{f} & \text{f} \\
\text{d} & \text{f} & \text{g}\n\end{array}$
 $\begin{array}{r} \n\text$

IX.2 The algorithm for evaluating $F(u)$

Let en nneyde en efnoon node odecope es es est n \bullet "ee \bullet popic" \bullet e \bullet

− ⁿ = jXn j=1 [P^j − P^j)] (9.27)

pnd n^o efncon ne 'yo \neq e \rightarrow epon y e Pt 'd '

 \bigwedge^2 e no e \bullet the second derivative of in the se dee $e \rightarrow n$ and $e \rightarrow m$ and considering the remainder of the series in \cdots aneoe eo nee a of M. Bony Teeoe oee e slightly s ook in a Bony s expansion in Bong s and r and r in s ed of j $\oint e$ y peep oee pto eeg nde ptypoo anoce apont ee and n lencopul oeped ppc on of e \overline{r} of to \overline{r} are arous peope functions \blacksquare to every everally cdn less nonsidering in considering \blacksquare pc y λn^* eoe λc ce $z n^*$ e

efe ence³

 \Box , B. Alpert. $p \neq p$ epert. Sparse on of \clubsuit ool the operators. Deperture \Box $n e \rightarrow y$

., CeLeen \mathbb{Z} dnd \mathbb{Z} and \mathbb{Z} algorithm for \mathbb{Z} for \mathbb{Z} and \mathbb{Z} $\rm p \quad c \ e \ \clubsuit \quad on$ ی SIAM Journal of Scienti c and Statistical Computing $\, \rm g \,$ Ye ne y fecnc ^{rep}ort, YALeB DO^Po^{rnic}e

 \vdots θ A. Co en D ec es nd C.

- \blacksquare , M \ulcorner \ulcorner \ulcorner e of feq ency channel decomposition of \ulcorner established eleteration of images and eleteration of images and electronical methods and electronical methods and electronical methods and electroni ode. The number of R and θ and θ courant θ and θ courant θ of M and θ courances. New Yo $n e \rightarrow y$
- \Box , Y. Meye. Le cc. \blacktriangleright en qe. e \blacktriangleright ondee e \blacktriangleright e. \Box e. \Box e. \Box e. \Box C^{π}MAD ne_de a^D pne
- \blacksquare Y. Meye nc pe d'ince de \clubsuit e enne et $\stackrel{\bullet}{\blacktriangle}$ e alg`ebres d'opère \clubsuit n w 6 ti $3T$