Feedback control stabilization of critical dynamics via resource transport on multilayer networks: How glia enable learning dynamics in the brain

I. INTRODUCTION

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B. Resource-transport dynamics

Resource diffuses between glia through their connection network (characterized by the adjacency matb) and between glia and the synapses they serve (via the glial-neural connection network characterized by the adjacency matbix Our model for the evolution of the amount of resourd the adjacency matbix glial cell i and the amount of resourd at synapse is

$$R_{i}^{t+1} = R_{i}^{t} + C_{1} + D_{G} \int_{j=1}^{T} U_{ij} R_{j}^{t} \check{S} R_{i}^{t}$$
$$+ D_{S} G_{i} R^{t} \check{S} R_{i}^{t} , \qquad (4)$$

$$R^{t+1} = R^{t} + D_{S} R^{t}_{i()} \check{S} R^{t} \check{S} C_{2} s^{t}_{m()}, \qquad (5)$$

where D_G is the rate of diffusion between glial cells, a D_G is the rate of diffusion between glia and synapses. Moreover, we enforce R_0 , i.e., if Eq. (5) yields $R^{t+1} < 0$, then we replace it by 0. The Prst term on the right hand side of (1), (R_i^t , is the amount of resource in glial cellat time t. The paramete C_1 denotes the amount of resource added to each glial cell at each time step (e.g., supplied by capillary blood vessels). For simplicity, we assume each glial cell has the same C_1 . The last two terms are the amount of resource transported to glial cell i, respectively, from its neighboring glial cells and from the synapses that it serves.

In Eq. (5), the Þrst term denotes the amount of resource at synapse at time t. The term proportional to

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