Feedback control stabilization of critical dynamics via resource transport on multilayer networks: How glia enable learning dynamics in the brain

> \mathbf{y}_1 , \mathbf{y}_2 , \mathbf{y}_3 , \mathbf{y}_4 , \mathbf{y}_5 , \mathbf{y}_6 , \mathbf{y}_7 , \mathbf{y}_7 , \mathbf{y}_8 , \mathbf{y}_7 , \mathbf{y}_8 , \mathbf{y}_9 , ¹University of Colorado at Boulder, Boulder, Colorado 80309, USA University of Arkansas, Fayetteville, Arkansas 72701, USA University of Colorado at Boulder, Boulder, Colorado 80309-0526, USA University of Maryland, College Park, Maryland 20742, USA $($ $)$ $)$ $($ $)$

Learning and memory are actually are actually changes in synapses. In the synapses in the simplest models, such models, such as \mathbf{r}_i synaptic potentiation typically leads to runaway excitation, but in reality there must exist processes that robustly p reserve overall stability of the neural system dynamics. How is this accomplished? Various approaches to this accomplished? In this accomplished? Various approaches to this accomplished? Various approaches to this appr $\mathbf{f}_\text{c} = \frac{1}{2} \left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)$ and $\mathbf{f}_\text{c} = \frac{1}{2} \left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)$ and $\mathbf{f}_\text{c} = \frac{1}{2} \left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)$

metabolic resources among the glia and, ultimately, to neural synapses where they are consumed. Our main result is that the biophysical constraints imposed by diffusive transport of metabolic resources through the glial net can prevent runaway growth of synaptic strength, both during ongoing activity and during learning. Our findings s st unappreciated role for glial transport of metabolites in the feedback control transport of metabolites in the feedback control stabilization of metabolites in the feedback control stabilization of metabolites in the neural network dynamics during learning learning learning learning learning learning learning learning.

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I. INTRODUCTION

 $\widehat{\mathbf{S}}_{\text{out}}$ of \mathbf{r}_{in} and diverse roles regulation \mathbf{r}_{in} in diverse roles regulating regulations. $\tau_{\rm c}$, $\tau_{\rm c}$ dividing $\tau_{\rm c}$ in $\tau_{\rm c}$, $\tau_{\rm c}$ learning-related changes in synapses \mathbf{r} . In this paper we have we have we have we have we have we have \mathbf{r} for the most important functions that f_{n+1} is thought to be been thought to be been thought to be a set of the best in the set of the best in the set of the best in the set of the set of the set of the set of the set served by the glial network \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i $T_{\rm eff}$ metabolic resources and neural synapses $T_{\rm eff}$. This is neglected in the neural synapses $T_{\rm eff}$. $\frac{m}{\sqrt{2}}$, $\frac{1}{2}$, $\frac{m}{\sqrt{2}}$, $\frac{1}{2}$, $\frac{m}{\sqrt{2}}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$ s_{max} showed that the glian form a bridge between the neural synapses in the neural synapses is synapses in the neural synapse and the brain variable \mathbf{r}_1 (b), \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4 \mathbf{r}_5 \mathbf{r}_6 $\mathbf{r}_{\mathbf{m}} = \mathbf{r}_{\mathbf{m}} + \mathbf{r}_{\mathbf{m}} + \mathbf{r}_{\mathbf{m}} + \mathbf{r}_{\mathbf{m}}$ more specifically and $\lim_{m\to\infty} \frac{1}{m} \lim_{n\to\infty} \frac{1}{m} \lim_{m\to\infty} \frac{1}{m} \lim_{n\to\infty} \frac{1}{m} \lim_{m\to\infty} \frac{1}{m}$ $\frac{d\mathbf{r}}{dt}$ on the synapses are $\frac{d\mathbf{r}}{dt}$ and $\frac{d\mathbf{r}}{dt}$ are $\frac{d\mathbf{r}}{dt}$ and $\frac{d\mathbf{r}}{dt}$ are $\frac{d\mathbf{r}}{dt}$ and $\frac{d\mathbf{r}}{dt}$ are $\frac{d\mathbf{r}}{dt}$ and $\frac{d\mathbf{r}}{dt}$ are $\frac{d\mathbf{r}}{dt}$ and $\frac{d$ $s_{\rm max}$ synapses, these resources diffuse through an extensive network and extensive networks. $\frac{1}{2}$. The biophysical properties of such diffusives of $\frac{1}{2}$ \mathbf{r}^{\top} , $\mathbf{r} = \mathbf{r}$, $\mathbf{r} = \frac{1}{2}$ fundamental influences \mathbf{r}^{\top} \overline{f} the dynamics of the activity of the activity of the network that \overline{f} consumes \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i are \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i active, and \mathbf{r}_i $s_{\text{max}} = \frac{1}{2} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=$ $\begin{bmatrix} 1 & \frac{1+\epsilon}{2} & \$ $d\mathbf{r}_i$, \mathbf{r}_i , \mathbf $f(x)$ is defined and differences and $f(x)$ in order to $f(x)$ study these possibilities, we introduce a computational model incorporation both a neural network and a glial network. Our model neurons interact via synapses whose effects \mathbf{r}_i and \mathbf{r}_i evolves \mathbf{r}_i evolves \mathbf{r}_i \mathbf{r}_i , then existing to activity-dependent learning rules, namely spin t_{max} time dependent plasticity (STP) [\[12,13\]](#page-6-0). Under many $\frac{1}{\sqrt{1-\mu}}$ circumstances, modeling of STP can result in unstable can result in $\frac{1}{\sqrt{m}} \sum_{i=1}^{n} \frac{1}{i} \sum_{i=1}^{n} \left[\frac{1}{i} \sum_{i=1}^{n} \frac{1}{i} \sum_{$

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types of learning rules to prevent such run and r discussion in the Conclusion section \mathcal{A} (13,14). T main finding of T or T o resources via the glian network can serve to prevent run \mathbf{r}_i $s_{\rm max}$ to Γ synaptic to Γ , thereby maintaining stability matrix Γ $\mathbf{r}^{\mathbf{r}}$ and \mathbf{r} defines the show that the this phenomenon that the this phenomenon that the show r_{π} r $\left\langle \mathbf{r}, \mathbf{r}, \mathbf{r}, \mathbf{r}, \mathbf{r} \right\rangle$, locally confined the glia; locally confined the glia; locally confined to \mathbf{r} p resources results p and p consumption of r of r' , r' neural network dynamics. The known roles played by the glian roles played by the glian roles played by the gli in synaptic plasticity are diverse and numerous $\int_{0}^{1} \frac{m}{n} \mathbf{r} \times \mathbf{r} \cdot \mathbf{r}$, to the diverse and $\int_{0}^{1} \frac{1}{n} \mathbf{r} \cdot \mathbf{r} \cdot \mathbf{r}$ our \mathbf{r} , \mathbf{r} is the first to show that metabolic the first that metabolic that metabolic show that \mathbf{r} r_{c} , and r_{c} such a stabilizing role. The stabilizing role of r_{c} such a stabilizing role. \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} are many examples of dynamical \mathbf{r} $\mathbf{p}^{\mathbf{w}}$, it is not need the matrix in \mathbf{r} in \mathbf{r} $\begin{CD} \mathbf{r} = \mathbf{r} - \mathbf{r$ interactions between the network \mathbf{r} increased, including is increased, including its increased, including its $s \sim \frac{1}{2}$ r_{onization} $\frac{1}{2}$, $\frac{1}{2}$ $\frac{1}{2}$, and $\frac{1}{2}$, $\frac{1}{2}$ some important cases, it is desirable that it is desirable that it is desirable that it is desirable that it i $\begin{array}{c} \mathbf{r}_{1} & \mathbf{r}_{2} & \mathbf{r}_{3} & \mathbf{r}_{4} & \mathbf{r}_{5} & \mathbf{r}_{6} & \mathbf{r}_{7} & \mathbf{r}_{8} & \mathbf{r}_{9} & \mathbf{r}_{1} & \mathbf{r}_{1} & \mathbf{r}_{1} & \mathbf{r}_{2} & \mathbf{r}_{3} & \mathbf{r}_{1} & \mathbf{r}_{2} & \mathbf{r}_{3} & \mathbf{r}_{4} & \mathbf{r}_{5} & \mathbf{r}_{6} & \mathbf{r}_{7} & \mathbf{r}_{8} & \mathbf{r}_{9} & \mathbf{r}_{$ for $\mathbf{r} = \begin{bmatrix} \mathbf{r} & \mathbf{r} & \mathbf{r} \\ \mathbf{r} & \mathbf{r} & \mathbf{r} \end{bmatrix}$ \mathbf{t}_1 that operating at the \mathbf{t}_2 provides the network \mathbf{t}_2 provides the network \mathbf{t}_3 with \mathbf{A}^{\dagger} is the different of the number of different, useful different, useful different, useful different, useful different, useful different of different of the different of the different of the different of t attractors, but with but with the perturbation to perturbation to perturbations of \mathbf{r}_1 18, for $r_{\rm in}$, it has been hypothesized that operating at critical point where the strengths of inhibitory and excitations are balanced provides are balanced provides \mathbf{r}_i information processing and storage and storage and storage and storage and neuronal networks in \mathbf{r} $m \sim 1$ and $r \sim 1$ in coarser models based on synchronization synchronization on synchronization on synchronization $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$; $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$; it has networks, it is it has networks, it is it has networks. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ in the phase transition in the phase transiti connectivity can minimize costs while achieving operational r_{eff} $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 1 \end{b$ can referred maintain operation at the critical point \mathbf{r}_i

 $\sqrt{7}$, $\sqrt{7}$, $\sqrt{7}$

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B. Resource-transport dynamics

Resource diffuses between glia through their connection network (characterized by the adjacency matub) and between glia and the synapses they serve (via the glial-neural connection network characterized by the adjacency matrix Our model for the evolution of the amount of resouR^teat glial cell i and the amount of resoured at synapse is

$$
R_{i}^{t+1} = R_{i}^{t} + C_{1} + D_{G} \bigcup_{j=1}^{T} U_{ij} R_{j}^{t} \check{S} R_{i}^{t}
$$

+
$$
D_{S} \bigcup_{j=1}^{M} G_{i} R_{i}^{t} \check{S} R_{i}^{t}, \qquad (4)
$$

$$
R^{t+1} = R^t + D_S R^t_{i(1)} \check{S} R^t \check{S} C_2 s^t_{m(1)},
$$
 (5)

where D_G is the rate of diffusion between glial cells, and is the rate of diffusion between glia and synapses. Moreover, we enforce R 0, i.e., if Eq. $\overline{6}$ yields R^{t+1} < 0, then we replace it by 0. The Þrst term on the right hand side of $E(q)$. (R_i^f , is the amount of resource in glial cellat time t. The paramete C_1 denotes the amount of resource added to each glial cell at each time step (e.g., supplied by capillary blood vessels). For simplicity, we assume each glial cell has the same C₁. The last two terms are the amount of resource transported to glial celli, respectively, from its neighboring glial cells and from the synapses that it serves.

In Eq. (5), the Þrst term denotes the amount of resource at synapse at time t. The term proportional to

$\sqrt{7}$, $\sqrt{7}$, $\sqrt{7}$

 \times 10⁴

recently by \mathbf{r} and \mathbf{r} are et al. 14. Reference for \mathbf{r} and $\mathbb{P}_{\mathbf{a}}$ modification of the STP rule that depends on a global depends $\frac{1}{2}$ metabolic resource that is dependent of the globally averaged by the globally averaged by the globally averaged by network activity and finds that it results in stability \mathbf{r}_1 In contrast to globally based resource regulation, we consider r $\mathbf{r} = \mathbf{r} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{2} \sum_{n=1}^{\$ which resource diffusion through the glial cell network plays $\mathbf{t} = \mathbf{t} + \mathbf{r}$ role in stabilizing the stabilizing the balanced dynamics at the balanced s_{12} , $\left(\begin{array}{c} 1 \\ 1 \end{array} \right)$ in Γ_{m} in the limit of Γ_{m} in the limit of Γ_{m} in the limit of Γ_{m} d_{c} , the resource at every glial cell would be resource at $\frac{1}{2}$ same, and our model would reduce to the one similar terms of the one similar terms of the one similar terms of $\left| \begin{array}{ccc} \cdots & 1 \end{array} \right|$. $\left| \begin{array}{ccc} \cdots & \cdots \end{array} \right|$, $\left| \begin{array}{ccc} \cdots & \cdots \end{array} \right|$, if the network of $\int_{0}^{1} \int_{0}^{1} \sin \theta \sin \theta \sin \theta \sin \theta$ to study the effect of different forms of $r_{\rm max}$ of $r_{\rm max}$ structured for spatially structured for spatial neuron-glia interactions. Finally, we note that we note that which we note that which we note that while in this contribution of the second state of the second state of the second state of the second state of the second s paper we use the neuronal network for the neuronal network, the neuronal neuronal neuronal neuronal neuronal ne $\begin{bmatrix} \mathbf{I}_{\text{out}} & \mathbf{I}_{\text{out$ $\frac{1}{2}$ extended to include modeling. The include modeling $\frac{1}{2}$

 $\mathbf{w}^T = \mathbf{v}^T = \mathbf{w}^T = \mathbf{v}^T = \mathbf{$ $0, 0, 11 - .000, 057 - .000110, 75.00015$

- \prod d. \prod_{m} \prod_{m} , \prod_{m} Γ Restriction Causes Cease \ldots \ldots \ldots \ldots \ldots \ldots [112](https://doi.org/10.1103/PhysRevLett.112.138103), 1¹ TO (201). $\left[\begin{matrix}1 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right]$
- \mathbf{H}^{H} , \mathbf{H}^{H} \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} , \mathbf{r} (00).
- $[30]$ F. A. C. Azevedo et al., \mathbf{r}_{max} numbers of neuronal and \mathbf{r}_{max} nonneuronal cells make the human brain $\frac{1}{\sqrt{2}}$. $\frac{1}{\sqrt{2}}$
- $\left[1\right]$ d. $\left[1\right]$ d. $\left[1\right]$ immunoreactive neurons in $\left[1\right]$ in \left $r_{\rm s}$, $v_{\rm t}$, $r_{\rm s}$, $\rm r_{\rm s}$, $v_{\rm r}$, $r_{\rm s}$, 261 , (1°) . [32] D. B. Larremore, W. L. Shew, and J. G. Restrepo, Predicting
	- Γ_{Cyl} and Γ_{Cyl} and Γ_{Cyl} and Γ_{Cyl} and Γ_{Cyl} and Γ_{Cyl} \mathcal{L} , Phys. Rev. Rev. Rev. [106](https://doi.org/10.1103/PhysRevLett.106.058101), 05 101 [\(2011\)](https://doi.org/10.1103/PhysRevLett.106.058101).
- $\frac{1}{3}$ S. S. P. P. Tang, S. S. Jiang, S. Jiang, S. Jiang, S. Zhang, and Z. Zhang, and Z. Zheng, A. Zheng, Howard, Howard, S. Zheng, S. Zheng, A. Zhe $\mathbf{t} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ range of excitations of $\mathbf{t} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\frac{1}{2}$, $\frac{1}{2}$
- $\frac{1}{2}$ or $\frac{1}{2}$ O. Kinouchi and M. Copelli, Optimal dynamical range of $\frac{1}{2}$ $\frac{1}{\|x\|^{1/2}}\leq \frac{1}{\|x\|^{1/2}}\sum_{\substack{r\in\mathbb{N},$