

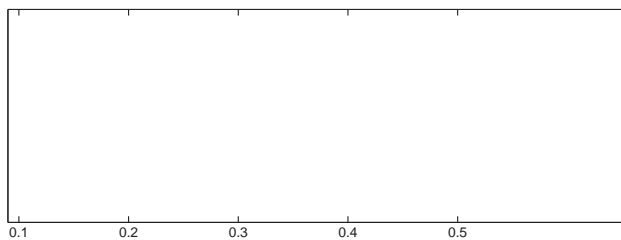
thereby eliminate one of the two equations for the center. The modulation system is thus

$$\frac{d}{dt} = \frac{2(\gamma + h_0)}{4} \int_{|x| < \xi} \text{sech}^2(|x - x_0| \xi^{-1}) dx, \tag{15}$$

$$\frac{dx_0}{dt} = \xi \int_{|x| < \xi} \text{sech}^2(|x - x_0| \xi^{-1}) \frac{x - x_0}{|x - x_0|} dx. \tag{16}$$

Note that when  $h_0 = 0$  and  $\gamma = 0$ , the remaining ODE  $\dot{x} = \dots$  agrees with the result in Ref. 23 and the more general result for solitons of nontrivial topological charge in Ref. 24. Equations (15) and (16) do not depend upon the slowly varying phase  $\theta_0$  so that the inclusion of long-range magnetostatic effects will lead to the same frequency shift given in Eq. (13), decoupling from the ODEs (15) and (16). The fixed points of this system correspond to steady-state conditions where there is a balance between uniform damping and localized spin torque, i.e., a dissipative droplet soliton. A fixed point at  $(\dot{x}, x_0) = (0, 0)$  leads to a relationship between the sustaining current and precession frequency

$$\dot{x} = \frac{2(\gamma + h_0)}{1 + \ln \text{sech} \xi^{-1} + 2 + \tanh \xi^{-1}}. \tag{17}$$







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