

# Normative decision rules in changing environments

Nicholas W Barendregt<sup>1\*</sup>, Joshua I Gold<sup>2</sup>, Krešimir Josi<sup>3</sup>, Zachary P Kilpatrick<sup>1</sup>

<sup>1</sup>Department of Applied Mathematics, University of Colorado Boulder, Boulder, United States; <sup>2</sup>Department of Neuroscience, University of Pennsylvania, Philadelphia, United States; <sup>3</sup>Department of Mathematics, University of Houston, Houston, United States

Abstract 🖏 🏚 👔 🦽 🦛 🦔 ma ll a la\_ a ma 🍋 🔎 l an 🐧 Ø 7.0 an, em c 1, en . . , ≛ mje Ita \_scall\_, ,● ♪ m l , n n m,o na al≛c, \_namcc,on alc\_n\_\_man lanz ,e c≛,en≛,em ,entit nela.V <u>, 1</u> c≛,enme In c a 🄊 🖲 ma no ac m la,on ∎ a<u></u>ta\_l ng nang 🔎 🖄 a,ec l a a ,● c a,●n n c 1 ,on 📬 la al., 2015), ac m la \_namcc, en , en 1 n I\_n 1 , ee . n ( n a 🄊 ma me clent at enc an incent, c n alc an 🛓 n al 🤳 Ø c,• 1,• algi cl.•n.n lcall, ,• max ( a a )lmam. n man, ctonmo t. Vto a to clin line 1 ac 1 1 1 ŀ al t c c 1 o m ,● al m,● 上 a С ກ 🔎ກ ลฑ ŀ С Ø C .• 🤊 àŋ, 🔎 m 🔎 🏄 🛴 nmlmn c L\_(∞•¹ L). ♥ me 🖡 n ลฑ ааа m a clenme l a a <u>هر 1 اهر 1 -</u> ,• m ,• c 🏘 an 🗢 🔹 🗴 🚽 man 🌵 👧 🖞 🏓 nac 🇢 n 🐧 🔎 m ∮, na al ⊾ m - a 🔍 ท 1111 а. ln⊾nje ma ลก ก al c 🖢 🔊 - ma 🛛 🗃 al .an a 🔎 at \_\_\_\_\_\_namc, a a ,● 批上 a a an 🖞 c a , en , an 🐧 🔎 a en an cemm mn. 7 .

\*For correspondence: nicholas.barendregt@colorado. edu

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## **Editor's evaluation**

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## Introduction

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$$y_n = \ln \frac{\Pr(s_+ |\xi_{1:n})}{\Pr(s_- |\xi_{1:n})} = \ln \frac{f_+(\xi_n)}{f_-(\xi_n)} + y_{n-1}.$$
 (2)

 $V(p_n; \rho) = \max\{V_+(p_n; \rho), V_-(p_n; \rho), V_w(p_n; \rho)\}$ 

= max

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$$R_c(t) = (R_2 - R_1)H_\theta(t - 0.5) + R_1.$$
(5)

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$$\mu(t) = (\mu_2 - \mu_1)H_{\theta}(t - 0.5) + \mu_1.$$
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## Discussion

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net\_ne ma me l nen-mene, enc cten telt av noelle le tilte a alet tan mant jottaa an et ne na ctentag tn la let mi oten nenn nt.

່ງໄປເລກa minalປາກ່ຽງລ່ຽ ກ່ອນ ໂຼງ ອຸກາດ ເປັງອກ-ma⊾ຫຼາ talat, menta ane ma a a titan alla ettati a amt,e la L‡m latut.,e am l, a cang atut c ,e ct. the a almot, ic al a n l a n ali () a i on n a c ali niti ac m la n i am o i (). In i i nito mo la i li o nom m n al igncan l i n i an offil a oi o c n C 🤊 lijcaa.c,en, ng,e ,e⊾anc,ent g lint ….e ,e max ,e ,e ) || | ,• m,● k ,•c an at⊾aam I( •7 •7 g,ent,e at⊾a am ta meta met nal,em,et alti⊾t,ent manc, ec., an moon all, pour pour bar a pool at 🖞 👘 🜒 a al **a** m 🤊 an e elenie noma. na lie namceen abitito abitine no a a cantin no a con no nititatione min c l plaelall, na ,eatitn ,e n ,, m,an la,ent nat⊾\_namctan ct,en lt.

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al<sup>t</sup> c<sup>t</sup>lul, ono lonatal<sup>t</sup>ag, n omaga<sup>t</sup> no oal( an n'a Lonamona-noma , '-noma , an iciagi.n **)** 1 aa, mnal<sup>is</sup>ta i<sup>s</sup>tan cla<sup>t</sup>wecent cajala... , ot in anal<u>t</u>™ of a analt Maria a seata seata seata a seata a seata seata a seata a seata a seata seata a seata a seata s ter a a a clatter molt ( ., ), tomollata call I en total mente totac m la ran mar ctent. cat l,⊕n n all a e clont a th o call in 111, a no ma anal t can o I a. In Iana, Ite Ilali n Jea a la sem can a li se nanye ma an t`c mye t`.,e am I, anye ma anal<u>t</u>tije ,e mana ,en a \_namc a at , c t t, el \_namcttmla, e , n a l a t, e l ກ l a c ໍt ເ,entan ໍt,el (e g ກ \_tg nal)m,e l.€em ກ g t m,e l-at n beja,ent me l- a ,ea b,bc at a -b,e ,en ,e....( )canabea n n sacommonal in comax an becia, nan comi m,e blabb ,e n ,e je m n⁵. mə در امرا ne, tanı a مرمور a la مرمور a la مرمور a məlc

$$V(p_{n}; \rho) = \max\{V_{+}(p_{n}; \rho), V_{-}(p_{n}; \rho), V_{w}(p_{n}; \rho)\}\$$
  
=  $\max\begin{cases}R_{c}p_{n} + R_{i}(1 - p_{n}) - t_{i} \ \rho, \\R\end{cases}$  choose  $s_{+}$ 

SNR-change task thresholds

• e ang at  $\nu$ , alle at  $\nu$  c  $|_m = \frac{2}{2}$ , a a at a at a all  $ma\nu$   $ma\nu$   $\mu(t)$  a m - n n t m on n

$$\mu(t) = (\mu_2 - \mu_1)H_{\theta}(t - 0.5) + \mu_1.$$

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Author contributions c lat W a n g , Com M . lg V g - og nal Za a klack, ton,	al a con, e a , e malanal št, ກ່ຽງa en, Vital a en, a , Vital a an ຮູບອີ a ຈີຍໄ, K em ເອັດ ກ່ຽງ a ໂດກ, Vital an ຮູ
Author ORCIDs c lat a n b b //ec t a tol to //ec lo //oc K K lo //ec / /00 Za a K lac to //	<pre>/0000-0002-32 %-942 00-0002- 01%-04%3 00-0002-1.9 \$-3.913 % /0000-0002-2%3\$-941</pre>
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Data availability Na c, o li , o n a all a , o m li , ¢, o _ac a	ీlీan g ీీaalala ీ/g c,em/n a ng/ aీ 1 29 8a3 9≸a3 989a0094a99 3414 9 4a2).
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