Write your name below. This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You are allowed to use two pages of notes (one piece of paper, front and back). You are not allowed to use a calculator or any computational software.

Name: Section:
(Chi/9:05/001; Grooms/11:15/002; Grooms/1:25/003)

1. (28 points: 4 each) If the statement is always true mark \TRUE"; if it is possible for the statement to be false then mark \FALSE". No justi cation is necessary .

_____(a) The polynomials $1 - x + 3x^2; x + x^2; x^2$ span the space of polynomials of degree at most 2.

_____(b) The columns of a matrix A are linearly independent if and only if the only solution to the homogeneous linear system $Ax = 0$ is the trivial one $x = 0$.

_____(c) Let $v_1; ::: ;$

2. (32 points, 8 each) Consider the following matrix

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 2 & 2 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$

(a) What is a basis for range(A)? What is its dimension? Explain your answer.

(b) What is a basis for corange(A)? What is its dimension? Explain your answer.

(c) What is a basis for kernel(A)? What is its dimension? Explain your answer.

(d) What is the dimension of cokernel(A)? Explain your answer.

3. (12 points, 6 each) Find the matrix form of the linear transformation

$$L\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - 4y \\ -2x + 3y \end{pmatrix}$$

(a) with respect to the standard basis of $\mathbb{R}^2$: $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and

(b) with respect to the basis of $\mathbb{R}^2$: $v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$; $v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

4. (28 points, 7 each points) Let $\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 9 \\ 0 \end{pmatrix}$.

   (a) Find all vectors that are orthogonal to both $\mathbf{v}$ and $\mathbf{w}$ when orthogonality is defined with respect to the dot product.