3. (20 pts) Do the following integrals converge or diverge? Evaluate the convergent integrals.

(a) $$\int_0^1 \frac{2e^{2x}}{1+e^{4x}}\,dx$$

(b) $$\int_2^1 \frac{1+\cos^2 x}{x \quad 1}\,dx$$

a Q H m i B Q M ,

(a) By direct calculation, we have

$$\begin{aligned}
\int_0^1 \frac{2e^{2x}}{1+e^{4x}}\,dx &= \lim_{t\,!\,1} \int_0^t \frac{2e^2x}{1+(e^{2x})^2}\,dx \qquad\qquad u = e^{2x};\, du = 2\,e^{2x}\,dx \\
&= \lim_{t\,!\,1} \int_0^{e^{2t}} \frac{1}{1+u^2}\,du \\
&= \lim_{t\,!\,1} \arctan(u)\Big|_1^{e^{2t}} \\
&= \lim_{t\,!\,1} \arctan(e^{2t}) \quad \frac{}{4} \\
&= \boxed{\frac{}{4}:}
\end{aligned}$$

Since the limit is fnite, the integral converges.

(b) We suspect that this integral diverges (since we are dividing by x) so let's try to show that via the comparison test. We can fnd a useful inequality as

$$\frac{1+\cos^2 x}{x \quad 1} \qquad \frac{1}{x \quad 1} \qquad \frac{1}{x}:$$

Since $\int_2^1 \frac{1}{x}\,dx$ is a divergent p-integral (i.e. $p = 1 \quad 1$), the Comparison test tells us that $\int_2^1 \frac{1+\cos^2 x}{x \quad 1}\,dx$ is also divergent.

4. (20 pts) Let R be the region bounded by $y = 1 \quad x^2$ and $y = x \quad 1$.

(a) Sketch and shade the region R. Label all axes, curves, and intersection points.

(b) Set up, but do not evaluate , integrals to determine each of the following:

i. The area of R using integration with respect to x.

ii. The area of R using integration with respect to y.

Solution:

(a) Graphing our equations, intersection points, and shading R, gives us

(b) Using the graph, our integrals are as follows:

i. From the graph, our integral is

$$\boxed{A = \int_{2}^{1} (1 \quad x^2) \quad (x \quad 1)\, dx:}$$

ii. Rewriting our equations in terms of y gives us $x = \sqrt{1 \quad y}$ and $x = y + 1$. Then, using our graph, we can compute the area as

$$\boxed{A = \int_{3}^{0} (y + 1) \quad (\sqrt{1 \quad y})\, dy + \int_{0}^{1} \sqrt{1 \quad y} \quad (\sqrt{1 \quad y})\, dy:}$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos 2x) \quad \sin^2 x = \frac{1}{2}(1 \quad \cos 2x) \quad \sin 2x = 2 \sin x \cos x \quad \cos 2x = \cos^2 x \quad \sin^2 x$$

A M p 2 ‘ b 2 h‘ B ; Q M Q K 2 i ‘ B + A M i 2 ; ‘ H A / 2 M i B i B 2 b

$$\int \frac{du}{p\overline{a^2 \quad u^2}} = \sin^{1} \frac{u}{a} + C; u^2 < a^2$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{1} \frac{u}{a} + C$$

$$\int \frac{du}{u\, p\overline{u^2 \quad a^2}} = \frac{1}{a} \sec^{1} \frac{u}{a} + C; u^2 > a^2$$

J B / T Q B M i _m H 2

$$\int_a^b f(x)\,dx \qquad x[f(\overline{x}_1) + f(\overline{x}_2) + \quad + f(\overline{x}_n)]; \quad x = \frac{b \quad a}{n}; \ \overline{x}_i = \frac{x_{i \ 1} + x_i}{2}; \ jE_M j \quad \frac{K(b \quad a)^3}{24n^2}$$

h\` T 2 x Q B / H _m H 2

$$\int_a^b f(x)\,dx \qquad \frac{x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \quad + 2f(x_{n \ 1}) + f(x_n)]; \quad x = \frac{b \quad a}{n}; \ jE_T j \quad \frac{K(b \quad a)^3}{12n^2}$$