1. (32 pts) The shaded region R_1, shown at right, is bounded by $y = \sqrt{x}\ln x$, $y = \ln(\sqrt{x})$, and the line $x = e^2$ in the first quad-

2. (14 pts) Find the length of the curve $y = \sqrt{4 - x^2}$, $0 \le x \le \frac{1}{2}$, by evaluating an integral.

Solution:

$$y = \sqrt{4 - x^2}$$

$$y' = -\frac{2x}{2\sqrt{4 - x^2}} = -\frac{x}{\sqrt{4 - x^2}}$$

$$\begin{aligned} L &= \int_a^b \sqrt{1 + (y')^2}\, dx = \int_0^{1/2} \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx \\ &= \int_0^{1/2} \sqrt{\frac{4}{4 - x^2}}\, dx = \int_0^{1/2} \frac{2}{\sqrt{4 - x^2}}\, dx \\ &= 2\sin^{-1}\left(\frac{x}{2}\right)\Big|_0^{1/2} = \boxed{2\sin^{-1}\left(\frac{1}{4}\right)} \end{aligned}$$

applying the $\sin^{-1}(x)$ antiderivative formula.

3. (14 pts) Solve the differential equation for y

5. (14 pts) Consider the geometric series $\frac{2}{3} + \frac{2m}{9} + \frac{2m^2}{27} + \frac{2m^3}{81} + \cdots$.

(a) For what values of m will the series converge?

(b) Can the sum of the series equal $\frac{2}{5}$? If so, find the corresponding value of m.

Solution:

(a) The series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3}\left(\frac{m}{3}\right)^{n-1}$