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An approximate renormalization for the break-up of invariant tori with three frequencies

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Abstract

Renormalization theory provides a description of the destruction of invariant tori for Hamiltonian systems of $1\frac{1}{2}$ or 2 degrees

dent resonances for ω , then $\omega = p$ where p is integral (remember the length of ω is unimportant). A frewhency ω is Diophantine if there is a $K \neq 0$ and $\tau > 2$ tersection of each pair of resonances defines rational frequencies $p_1 = [1, 0, 0]$, $p_2 = [0, 1, 0]$, $p_3 = [0, 0, 1]$. The frequencies *n* also delineate the cone: it is the

such that $\forall m \in \mathbb{Z}^3 \setminus 0$, $|m \omega| / |\omega| > K / |m|^{\tau}$.

When A=B=C=0, the momenta (u, v) are constant in time and every orbit lies on a three torus. If $\omega(u, v)$ is incommensurate, the orbit densely covers the torus. If ω is Diophantine, then the KAM theorem implies that there is a torus with this frequency for small values of the amplitudes. We are interested convex hull of the three vectors. We denote the cone by either of the matrices

$$M = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}, \quad P = (p_1, p_2, p_3) \; .$$

We assume ω is inside the cone, i.e. $\omega_i \ge 0$.

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$$\begin{pmatrix} \delta k' \\ \delta l' \end{pmatrix} = \begin{pmatrix} -1 & \sigma \\ -\sigma^{-4} & 0 \end{pmatrix} \begin{pmatrix} \delta k \\ \delta l \end{pmatrix}$$

The eigenvalues are

 $\lambda = \sigma^{-3/2} e^{\pm i\psi/2},$ $\cos(\psi) = \frac{1}{2}(\sigma - 1), \quad \psi \approx 2\pi \times 0.22404487.$ (9)

The mass renormalization is a linear map. Recall that it has been constructed to preserve the subspace $\alpha\gamma - \beta^2 = 1$. Since the wavenumber map is contracting, we can evaluate the mass map at the fixed point $k = \sigma^{-1}$. This gives the eigenvalues

$$\begin{pmatrix} a' \\ b' \\ c' \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} \log(\sigma^8 \beta/2) \\ \log(\sigma^3) \\ \log(\sigma^3) \end{pmatrix}.$$
(13)

Thus stability is governed by the linear matrix above. This matrix has characteristic polynomial $\lambda^3 - \lambda^2 - 1 = 0$ (interestingly, this polynomial is not related to the spiral mean), so that

$$\lambda_1 = \delta \approx 1.465571232,$$

$$\lambda_{2,3} = \delta^{-1/2} e^{\pm 11\,856478541} \,. \tag{14}$$

(10)

and a two dimensional, spiral stable manifold. The

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