

An approximate renormalization for the break-up of invariant tori The Kolmogorov-Amol'd-Moser (KAM) theoand (particularly for two degrees of freedom) the loss of freedom) the loss of freedom) the loss of $\mathcal{O}(n)$ α stability. The stability is of α is of α interest to develop α

 R_{c} M_{c} V_{c} in N_{c} in $\frac{1}{2}$ I_{c} N_{c} $\frac{1}{2}$ iant toil of $\frac{1}{2}$ 1985 , J. Stark

Centre for Nonlinear Dynamics and Applications, University College London, London WC1E 6BT, UK

Received 8 April 1994, revised manuscript received 21 April 1994, accepted for publication 17 May 1994 communicated by A.R. Bishop theory to show that for a critical invariant converse-

Abstract

no invariant to invariant to those of the continuously deformable to those of the continuously defined to thos

tion and the set for which the set for which the set of invariant torus of that frequency are both open. The

i Mathematics Department, University of Warwich Oriental Coventry, University of Warwich Oriental Coventry, Co
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Renormalization theory provides a description of the destruction of invariant tori for Hamiltonian systems of $1\frac{1}{2}$ or 2 degrees find a similar self-similarity for the breakup oftori in

> higher \sim 10-15]. In the authors studied three frequency systems: etther maps of the toms, volume preserving maps, or four dimen-

sional symplecular maps in the symplecular maps of the symplecular maps in the

dent resonances for ω , then $\omega = p$ where p is integral (remember the length of ω is unimportant). A fre q *n*ency ω is *Dionhantine* if there is a $K \neq 0$ and $\tau > 2$

the toms. If to is Dlophantlne, then the KAM theorem implies that there is a torus wtth this frequency

in determining the parameters for which such a torus for which such a torus $\frac{1}{2}$

real transformations to coordinates that are more closely aligned with the incommensurate flow. We use tersection of each pair of resonances defines rational frequencies $p_1 = [1, 0, 0], p_2 = [0, 1, 0], p_3 = [0, 0,$ 11 The frequencies p , also delineate the cone; it is the

such that $\forall m \in \mathbb{Z}^3 \setminus 0$, $|m \omega|/|\omega| > K/|m|^{\tau}$.

When $A=B=C=0$, the momenta (u, v) are constant in time and every orbit lies on a three torus. If $\omega(u, v)$ is incommensurate, the orbit densely covers
the torus. If ω is Diophantine, then the KAM theo-Each of the three phases in *V(x, y, z)* corresponds to a sinal values of the amplitudes. We are interested onances matrix $\mathcal{O}(\mathcal{O})$ and $\mathcal{O}(\mathcal{O})$ and $\mathcal{O}(\mathcal{O})$. The following matrix $\mathcal{O}(\mathcal{O})$

Each resonance corresponds to a plane in R 3 or a line

Choose the new cone that contains ~0 and repeat this convex null of the three vectors, we define the cone by either of the matrices

To construct the Farey sequence for to, divide the

sponding resonance *m'=m~-m2.* There is now a nght and a left cone PR = (P3, Pl, P') and PL = (P3, PS, P') and PL = (P3, P3, P3, P3, P3, P3, P3, P3, P3, P

$$
M = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}, \quad P = (p_1, p_2, p_3) \; .
$$

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We assume ω is inside the cone, i.e. $\omega_i \ge 0$.

m.

$$
\begin{pmatrix} \delta k' \\ \delta l' \end{pmatrix} = \begin{pmatrix} -1 & \sigma \\ -\sigma^{-4} & 0 \end{pmatrix} \begin{pmatrix} \delta k \\ \delta l \end{pmatrix}
$$

The eigenvalues are

 $\lambda = \sigma^{-3/2} e^{\pm i \psi/2}$, $\cos(\psi) = \frac{1}{2}(\sigma - 1), \quad \psi \approx 2\pi \times 0.22404487$. (9)

The mass renormalization is a linear map. Recall that it has been constructed to preserve the subspace $\alpha \gamma - \beta^2 = 1$. Since the wavenumber map is contracting, we can evaluate the mass map at the fixed point $k = a^{-1}$. This gives the eigenvalues

$$
\begin{pmatrix} a' \\ b' \\ c' \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} \log(\sigma^8 \beta/2) \\ \log(\sigma^3) \\ \log(\sigma^3) \end{pmatrix}.
$$
 (13)

Thus stability is governed by the linear matrix above. This matrix has characteristic polynomial $\lambda^3 - \lambda^2$ $-1 = 0$ (interestingly, this polynomial is not related to the spiral mean), so that

$$
\lambda_1 = \delta \approx 1.465571232,
$$

$$
\lambda_{2,3} = \delta^{-1/2} e^{\pm i 1.856478541} \,. \tag{14}
$$

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 $\mathcal{B}=\mathcal{B}(\mathcal{B})$, the affine map $\mathcal{B}=\mathcal{B}(\mathcal{B})$, the affine map $\mathcal{B}(\mathcal{B})$

Ac- O-14fl, Bc= O-Sfl' CC= O. 1ift. (12)

 (10)

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